SPC 407
Sheet 2 - Solution
Compressible Flow - Governing Equations

1. Is it possible to accelerate a gas to a supersonic velocity in a converging nozzle? Explain.
   Solution:
   No, it is not possible.
   The only way to do it is to have first a converging nozzle, and then a diverging nozzle.

2. Consider a converging nozzle with sonic speed at the exit plane. Now the nozzle exit area is reduced while the nozzle inlet conditions are maintained constant. What will happen to (a) the exit velocity and (b) the mass flow rate through the nozzle?
   Solution:
   (a) The exit velocity remains constant at sonic speed
   (b) The mass flow rate through the nozzle decreases because of the reduced flow area.
   Without a diverging portion of the nozzle, a converging nozzle is limited to sonic velocity at the exit.

3. In March 2004, NASA successfully launched an experimental supersonic-combustion ramjet engine (called a scramjet) that reached a record-setting Mach number of 7. Taking the air temperature to be -20 C, determine the speed of this engine.
   Solution:
   Assumptions: Air is an ideal gas with constant specific heats at room temperature.
   Properties: The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.
   The temperature is -20 + 273.15 = 253.15 K. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(253.15\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 318.93\ \text{m/s}$$

and

$$V = c\mathrm{Ma} = (318.93 \text{ m/s})(7)\left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}}\right) = 8037 \text{ km/h} \cong \mathbf{8040\ km/h}$$

Note that extremely high speed can be achieved with scramjet engines. We cannot justify more than three significant digits in a problem like this.

4. Is it possible to accelerate a fluid to supersonic velocities with a velocity other than the sonic velocity at the throat? Explain

   Solution:

   No, if the flow in the throat is subsonic. If the velocity at the throat is subsonic, the diverging section would act like a diffuser and decelerate the flow. **Yes, if the flow in the throat is already supersonic**, the diverging section would accelerate the flow to even higher Mach number.

   In duct flow, the latter situation is not possible unless a second converging-diverging portion of the duct is located upstream, and there is sufficient pressure difference to choke the flow in the upstream throat.

5. Air enters a converging–diverging nozzle at 1.2 MPa with a negligible velocity. Approximating the flow as isentropic, determine the back pressure that would result in an exit Mach number of 1.8.

   Solution:

   Assumptions: 1. Air is an ideal gas with constant specific heats.
   2. Flow through the nozzle is steady, one-dimensional, and isentropic.

   Properties: The specific heat ratio of air is k = 1.4.

   The stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. It remains constant throughout the nozzle since the flow is isentropic,

   $P_0 = P_i = 1.2$ MPa

   From Table A-13 at $M_e = 1.8$, we read $P_e / P_0 = 0.1740$.

   Thus, $P = 0.1740\ P_0 = 0.1740\ (1.2 \text{ MPa}) = 0.209 \text{ MPa} = 209 \text{ kPa}$

   If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

6. Air enters a converging–diverging nozzle of a supersonic wind tunnel at 150 psia and 100 F with a low velocity. The flow area of the test section is equal to the exit area of the nozzle, which is 5 ft^2. Calculate the pressure, temperature, velocity, and mass flow rate in the test section for a Mach number Ma = 2. Explain why the air must be very dry for this application.

Solution:

Assumptions: 1. Air is an ideal gas.

2. Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties: The properties of air are k = 1.4 and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$P_0 = P_i$ = 150 psia and $T_0 = T_i$ = 100 F = 560 R

Then,

$$T_e = T_0\left(\frac{2}{2+(k-1)\mathrm{Ma}^2}\right) = (560\ \mathrm{R})\left(\frac{2}{2+(1.4-1)2^2}\right) = \mathbf{311R}$$

$$P_e = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (150\ \mathrm{psia})\left(\frac{311}{560}\right)^{1.4/0.4} = \mathbf{19.1\ psia}$$

$$\rho_e = \frac{P_e}{RT_e} = \frac{19.1\ \mathrm{psia}}{(0.3704\ \mathrm{psia.ft^3/1bm\cdot R})(311\ \mathrm{R})} = 0.166\ \mathrm{1bm/ft^3}$$

The nozzle exit velocity can be determined from $V_e = Ma_e c_e$, where c_e is the speed of sound at the exit conditions,

$$V_e = \mathrm{Ma}_e c_e = \mathrm{Ma}_e\sqrt{kRT_e} = (2)\sqrt{(1.4)(0.06855\ \mathrm{Btu/1bm\cdot R})(311\ \mathrm{R})\left(\frac{25{,}037\ \mathrm{ft^2/s^2}}{1\ \mathrm{Btu/1bm}}\right)} = 1729\ \mathrm{ft/s} \cong \mathbf{1730\ ft/s}$$

Finally,

$$\dot{m} = \rho_e A_e V_e = (0.\ 166\ \mathrm{1bm/ft^3})(5\ \mathrm{ft^2})(1729\ \mathrm{ft/s}) = 1435\ \mathrm{lbm/s} \cong \mathbf{1440\ lbm/s}$$

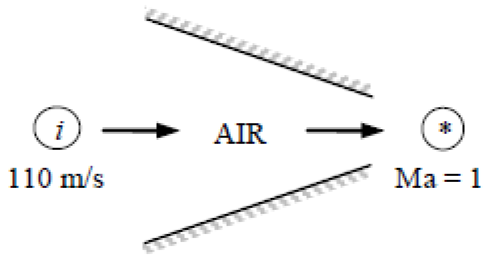
Air must be very dry in this application because the exit temperature of air is extremely low, and any moisture in the air will turn to ice particles.

7. Air enters a nozzle at 0.5 MPa, 420 K, and a velocity of 110 m/s. Approximating the flow as isentropic, determine the pressure and temperature of air at a location where the air velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area?

Solution:

Assumptions:

1. Air is an ideal gas with constant specific heats at room temperature.
2. Flow through the nozzle is steady, one-dimensional, and isentropic.



Properties: The properties of air are k = 1.4 and c_p = 1.005 kJ/kg·K.
The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *.
We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 420\,\text{K} + \frac{(110\,\text{m/s})^2}{2\times 1.005\,\text{kJ/kg}\cdot\text{K}}\left(\frac{1\,\text{kJ/kg}}{1000\,\text{m}^2/\text{s}^2}\right) = 426.02$$

and

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.5\,\text{MPa})\left(\frac{426.02\,\text{K}}{420\,\text{K}}\right)^{1.4/(1.4-1)} = 0.52554\,\text{MPa}$$

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma = 1, we read T/T_0 = 0.8333, P/P_0 = 0.5283.
Thus,

$$T = 0.8333T_0 = 0.8333(426.02\,\text{K}) = 355.00\,\text{K} \approx \mathbf{355\,K}$$

and

$$P = 0.5283P_0 = 0.5283(0.52554\,\text{MPa}) = 0.27764\,\text{MPa} \approx \mathbf{0.278\,MPa} = \mathbf{278\,kPa}$$

Also,

$$c_i = \sqrt{kRT_i} = \sqrt{(1.4)(0.287\,\text{kJ/kg}\cdot\text{K})(420\,\text{K})\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right)} = 410.799\,\text{m/s}$$

and

$$\mathrm{Ma}_i = \frac{V_i}{c_i} = \frac{110\text{ m/s}}{410.799\text{ m/s}} = 0.2678$$

$$\mathrm{Ma}_i = \frac{V_i}{c_i} = \frac{150\text{ m/s}}{410.799\text{ m/s}} = 0.3651$$

From Table A-13 at this Mach number we read $A_i/A^* = 2.3343$. Thus, the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A} = \frac{1}{2.3343} = 0.42839 \cong \mathbf{0.428}$$

We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

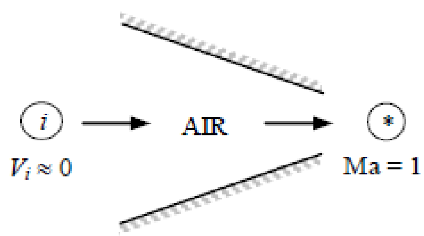
8. Repeat Prob. 7 assuming the entrance velocity is negligible.
Solution:
Assumptions:
1. Air is an ideal gas with constant specific heats at room temperature.
2. Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties: The properties of air are k = 1.4 and c_p = 1.005 kJ/kg·K.
The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i = 420\text{ K}$$

and

$$P_0 = P_i = 0.5\text{ MPa}$$



From Table A-13 (or from Eqs. 12-18 and
=0.8333, P/P_0 = 0.5283. Thus,

$$T = 0.8333T_0 = 0.8333(420\text{ K}) = \mathbf{350\ K}$$

and

$$P = 0.5283P_0 = 0.5283(0.5\text{ MPa}) = \mathbf{0.264\ MPa}$$

The Mach number at the nozzle inlet is Ma = 0 since Vi = 0. From Table A-13 at this Mach number we read Ai/A* = ∞ .

Thus, the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{\infty} = \mathbf{0}$$

If we solve this problem using the relations for compressible isentropic flow, the results would be identical.