SPC 407
Sheet 5 - Solution
Compressible Flow – Rayleigh Flow

1. Consider subsonic Rayleigh flow of air with a Mach number of 0.92. Heat is now transferred to the fluid and the Mach number increases to 0.95. Does the temperature T of the fluid increase, decrease, or remain constant during this process? How about the stagnation temperature T_0?

   Solution:
   In Rayleigh flow, the stagnation temperature T0 always increases with heat transfer to the fluid, but the temperature T decreases with heat transfer in the Mach number range of $0.845 < Ma < 1$ for air. Therefore, the temperature in this case will decrease.
   This at first seems counterintuitive, but if heat were not added, the temperature would drop even more if the air were accelerated isentropically from Ma = 0.92 to 0.95.

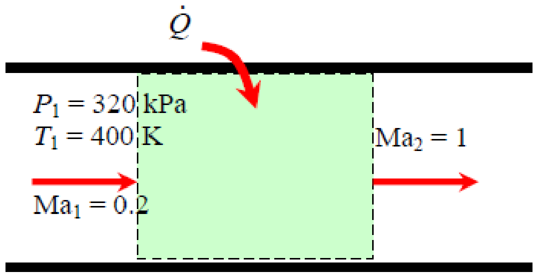
2. Consider subsonic Rayleigh flow that is accelerated to sonic velocity (Ma = 1) at the duct exit by heating. If the fluid continues to be heated, will the flow at duct exit be supersonic, subsonic, or remain sonic?

   Solution:
   The flow is choked, and thus the flow at the duct exit remains sonic.
   There is no mechanism for the flow to become supersonic in this case.

3. Argon gas enters a constant cross-sectional area duct at $Ma_1 = 0.2$, $P_1 = 320$ kPa, and $T_1 = 400$ K at a rate of 1.2 kg/s. Disregarding frictional losses, determine the highest rate of heat transfer to the argon without reducing the mass flow rate.

Solution:

Take the properties of argon to be k = 1.667, $c_p = 0.5203$ kJ/kg-K, and R = 0.2081 kJ/kg-K.



Heat transfer stops when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. The inlet stagnation temperature is

$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (400\,\mathrm{K})\left(1+\frac{1.667-1}{2}0.2^2\right) = 405.3\,\mathrm{K}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $\mathrm{Ma}_2 = 1$)

Therefore,

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\mathrm{Ma}_1^2[2+(k-1)\mathrm{Ma}_1^2]}{(1+k\mathrm{Ma}_1^2)^2} = \frac{(1.667+1)0.2^2[2+(1.667-1)0.2^2]}{(1+1.667\times 0.2^2)^2} = 0.1900$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.1900}$$

$$T_{02} = T_{01}/0.1900 = (405.3\,\mathrm{K})/0.1900 = 2133\ \mathrm{K}$$

Then the rate of heat transfer becomes

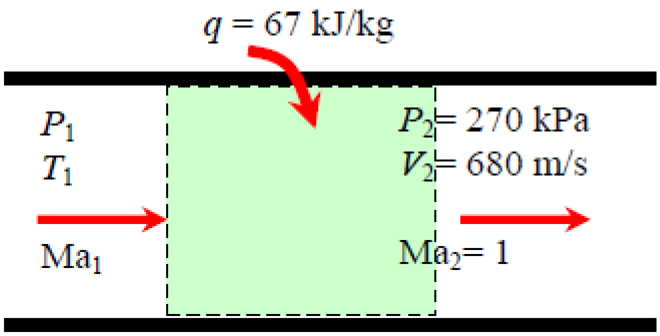
$$\dot{Q} = \dot{m}_{\mathrm{air}}c_p(T_{02}-T_{01}) = (1.2\,\mathrm{kg/s})(0.5203\,\mathrm{kJ/kg\cdot K})(2133-400)\,\mathrm{K} = \mathbf{1080\ kW}$$

It can also be shown that $T_2 = 1600$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-15 since they are based on k = 1.4.

4. Air is heated as it flows subsonically through a duct. When the amount of heat transfer reaches 67 kJ/kg, the flow is observed to be choked, and the velocity and the static pressure are measured to be 680 m/s and 270 kPa. Disregarding frictional losses, determine the velocity, static temperature, and static pressure at the duct inlet.

Solution:

Take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg-K, and R = 0.287 kJ/kg-K.



Noting that sonic conditions exist at the exit, the exit temperature is

$$c_2 = V_2/\mathrm{Ma}_2 = (680\ \mathrm{m/s})/1 = 680\ \mathrm{m/s}$$

$$c_2 = \sqrt{kRT_2} \rightarrow \sqrt{(1.4)(0.287\ \mathrm{kJ/kg \cdot K})T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 680\ \mathrm{m/s}$$

It gives T_2 = 1151 K. Then the exit stagnation temperature becomes

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 1151\ \mathrm{K} + \frac{(680\ \mathrm{m/s})^2}{2 \times 1.005\ \mathrm{kJ/kg \cdot K}}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 1381\ \mathrm{K}$$

The inlet stagnation temperature is, from the energy equation ($q = c_p(T_{02} - T_{01})$),

$$T_{01} = T_{02} - \frac{q}{c_p} = 1381\ \mathrm{K} - \frac{67\ \mathrm{kJ/kg}}{1.005\ \mathrm{kJ/kg \cdot K}} = 1314\ \mathrm{K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value in this case is T_{02} since the flow is choked. Therefore, T_0^* = T_{02} = 1381 K. Then the stagnation temperature ratio at the inlet, and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{01}}{T_0^*} = \frac{1314\ \mathrm{K}}{1381\ \mathrm{K}} = 0.9516 \qquad \rightarrow \qquad \mathrm{Ma}_1 = 0.7779 \cong \mathbf{0.778}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$Ma_1 = 0.7779$: $T_1/T^* = 1.018$, $P_1/P^* = 1.301$, $V_1/V^* = 0.7852$

$Ma_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $V_2/V^* = 1$

Then the inlet temperature, pressure, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{1.018} \quad \rightarrow \quad T_1 = 1.018T_2 = 1.018(1151\,\text{K}) = \mathbf{1172K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.301} \quad \rightarrow \quad P_1 = 1.301P_2 = 1.301(270\,\text{kPa}) = \mathbf{351.3kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{1}{0.7852} \quad \rightarrow \quad V_1 = 0.7852V_2 = 0.7852(680\,\text{m/s}) = \mathbf{533.9m/s}$$

Note that the temperature and pressure decreases with heating during this subsonic Rayleigh flow while velocity increases. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

5. Compressed air from the compressor of a gas turbine enters the combustion chamber at T_1 = 700 K, P_1 = 600 kPa, and Ma_1 = 0.2 at a rate of 0.3 kg/s. Via combustion, heat is transferred to the air at a rate of 150 kJ/s as it flows through the duct with negligible friction. Determine the Mach number at the duct exit, and the drop in stagnation pressure $P0_1$ - $P0_2$ during this process.

Solution:

Take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg-K, and R = 0.287 kJ/kg-K.

The inlet stagnation temperature and pressure are

$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (700\ \mathrm{K})\left(1+\frac{1.4-1}{2}0.2^2\right) = 705.6\ \mathrm{K}$$

$$P_{01} = P_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\ \mathrm{kPa})\left(1+\frac{1.4-1}{2}0.2^2\right)^{1.4/0.4}$$
$$= 617.0\ \mathrm{kPa}$$

The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\mathrm{air}} c_p (T_{02} - T_{01}) \quad\rightarrow\quad 150\ \mathrm{kJ/s} = (0.3\ \mathrm{kg/s})(1.005\ \mathrm{kJ/kg\cdot K})(T_{02} - 705.6)\ \mathrm{K}$$

It gives

T_{02} = 1203 K

At Ma_1 = 0.2 we read from T_{01}/T_0^* = 0.1736 (Table A-15). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{705.6\ \mathrm{K}}{0.1736} = 4064.5\ \mathrm{K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-15)

$$\frac{T_{02}}{T_0^*} = \frac{1203\ \mathrm{K}}{4064.5\ \mathrm{K}} = 0.2960 \quad\rightarrow\quad \mathrm{Ma}_2 = 0.2706 \cong \mathbf{0.271}$$

Also,

$$\mathrm{Ma}_1 = 0.2 \quad\rightarrow\quad P_{01}/P_0^* = 1.2346$$
$$\mathrm{Ma}_2 = 0.2706 \quad\rightarrow\quad P_{02}/P_0^* = 1.2091$$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.2091}{1.2346} = 0.9794 \quad\rightarrow\quad P_{02} = 0.9794 P_{01} = 0.9794(617\ \mathrm{kPa}) = 604.3\ \mathrm{kPa}$$

and

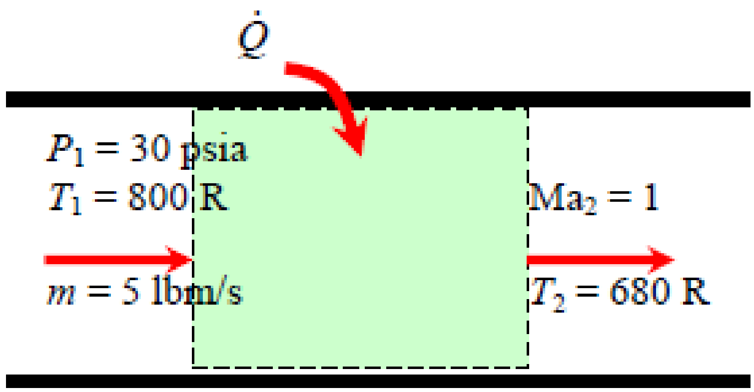
$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 604.3 = \mathbf{12.7\ kPa}$$

This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

6. Air flows with negligible friction through a 4-in diameter duct at a rate of 5 lbm/s. The temperature and pressure at the inlet are T_1 = 800 R and P_1 = 30 psia, and the Mach number at the exit is Ma_2 = 1. Determine the rate of heat transfer and the pressure drop for this section of the duct.

Solution:

Take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg-K, and R = 0.287 kJ/kg-K.



The inlet density and velocity of air are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{30 \text{ psia}}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(800 \text{ R})} = 0.1012 \text{ lbm/ft}^3$$

$$V_1 = \frac{\dot{m}_{air}}{\rho_1 A_{c1}} = \frac{5 \text{ lbm/s}}{(0.1012 \text{ lbm/ft}^3)[\pi(4/12 \text{ ft})^2/4]} = 565.9 \text{ ft/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 800 \text{ R} + \frac{(565.9 \text{ ft/s})^2}{2\times 0.2400 \text{ Btu/lbm}\cdot\text{R}}\left(\frac{1 \text{ Btu/lbm}}{25{,}037 \text{ ft}^2/\text{s}^2}\right) = 826.7 \text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855 \text{ Btu/lbm}\cdot\text{R})(800 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = 1386 \text{ ft/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{565.9 \text{ ft/s}}{1386 \text{ ft/s}} = 0.4082$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\text{Ma}_1 = 0.4082$: $T_1/T^* = 0.6310$, $P_1/P^* = 1.946$, $T_{01}/T_0^* = 0.5434$

$\text{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6310} \qquad \rightarrow \quad T_2 = T_1/0.6310 = (800\text{ R})/0.6310 = 1268\text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.946} \qquad \rightarrow \quad P_2 = P_1/2.272 = (30\text{ psia})/1.946 = 15.4\text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.5434} \qquad \rightarrow \quad T_{02} = T_{01}/0.1743 = (826.7\text{ R})/0.5434 = 1521\text{ R}$$

Then the rate of heat transfer and the pressure drop become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (5\text{ lbm/s})(0.2400\text{ Btu/lbm}\cdot\text{R})(1521 - 826.7)\text{ R} = \mathbf{834\ Btu/s}$$

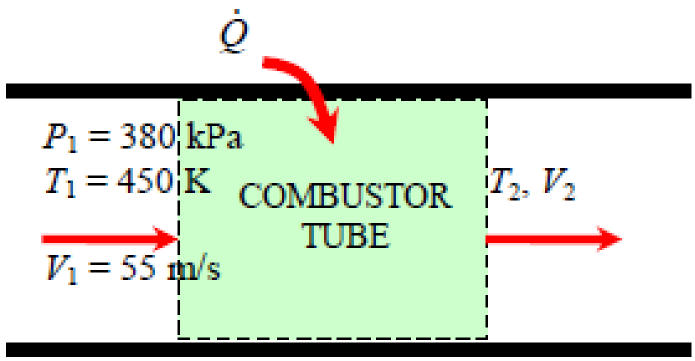
$$\Delta P = P_1 - P_2 = 30 - 15.4 = \mathbf{14.6psia}$$

Note that the entropy of air increases during this heating process, as expected.

7. Consider a 16-cm-diameter tubular combustion chamber. Air enters the tube at 450 K, 380 kPa, and 55 m/s. Fuel with a heating value of 39,000 kJ/kg is burned by spraying it into the air. If the exit Mach number is 0.8, determine the rate at which the fuel is burned and the exit temperature. Assume complete combustion and disregard the increase in the mass flow rate due to the fuel mass.

Solution:

Take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg-K, and R = 0.287 kJ/kg-K.



The inlet density and mass flow rate of air are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{380\text{ kPa}}{(0.287\text{ kJ/kgK})(450\text{ K})} = 2.942\text{ kg/m}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.942\text{ kg/m}^3)[\pi(0.16\text{ m})^2/4](55\text{ m/s}) = 3.254\text{ kg/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 450\text{ K} + \frac{(55\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 451.5\text{ K}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(450\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 425.2\text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{55\text{ m/s}}{425.2\text{ m/s}} = 0.1293$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15) (We used analytical functions):

$\text{Ma}_1 = 0.1293$: $T_1/T^* = 0.09201$, $T_{01}/T^* = 0.07693$, $V_1/V^* = 0.03923$

$\text{Ma}_2 = 0.8$: $T_2/T^* = 1.0255$, $T_{02}/T^* = 0.9639$, $V_2/V^* = 0.8101$

The exit temperature, stagnation temperature, and velocity are determined to be

$$\frac{T_2}{T_1}=\frac{T_2/T^*}{T_1/T^*}=\frac{1.0255}{0.09201}=11.146 \quad\rightarrow\quad T_2=11.146T_1=11.146(450\text{ K})=\mathbf{5016K}$$

$$\frac{T_{02}}{T_{01}}=\frac{T_{02}/T^*}{T_{01}/T^*}=\frac{0.9639}{0.07693}=12.530 \quad\rightarrow\quad T_{02}=12.530T_{01}=12.530(451.5\text{ K})=5658\text{ K}$$

$$\frac{V_2}{V_1}=\frac{V_2/V^*}{V_1/V^*}=\frac{0.8101}{0.03923}=20.650 \quad\rightarrow\quad V_2=20.650V_1=20.650(55\text{ m/s})=1136\text{ m/s}$$

Then the mass flow rate of the fuel is determined to be

$$q=c_p(T_{02}-T_{01})=(1.005\text{ kJ/kg}\cdot\text{K})(5658-451.5)\text{ K}=5232\text{ kJ/kg}$$

$$\dot{Q}=\dot{m}_{\text{air}}q=(3.254\text{ kg/s})(5232\text{ kJ/kg})=17{,}024\text{ kW}$$

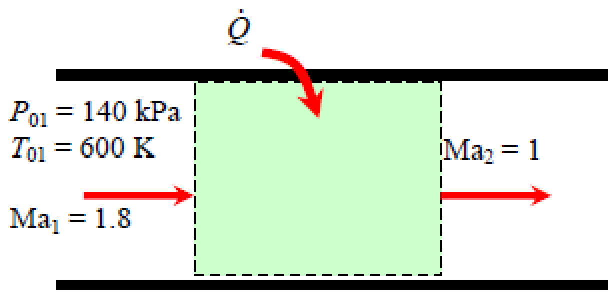
$$\dot{m}_{\text{fuel}}=\frac{\dot{Q}}{HV}=\frac{17{,}024\text{ kJ/s}}{39{,}000\text{ kJ/kg}}=\mathbf{0.4365kg/s}$$

Note that both the temperature and velocity increase during this subsonic Rayleigh flow with heating, as expected. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

8. Consider supersonic flow of air through a 7-cm-diameter duct with negligible friction. Air enters the duct at $Ma_1 = 1.8$, $P0_1 = 140$ kPa, and $T0_1 = 600$ K, and it is decelerated by heating. Determine the highest temperature that air can be heated by heat addition while the mass flow rate remains constant.

Solution:

Take the properties of air to be k = 1.4, $c_p = 1.005$ kJ/kg-K, and R = 0.287 kJ/kg-K.



Heat transfer will stop when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. Knowing stagnation properties, the static properties are determined to be

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (600\,\mathrm{K})\left(1+\frac{1.4-1}{2}1.8^2\right)^{-1} = 364.1\,\mathrm{K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (140\,\mathrm{K})\left(1+\frac{1.4-1}{2}1.8^2\right)^{-1.4/0.4}$$
$$= 24.37\,\mathrm{kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{24.37\,\mathrm{kPa}}{(0.287\,\mathrm{kJ/kgK})(364.1\,\mathrm{K})} = 0.2332\,\mathrm{kg/m^3}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\,\mathrm{kJ/kg\cdot K})(364.1\,\mathrm{K})\left(\frac{1000\,\mathrm{m^2/s^2}}{1\,\mathrm{kJ/kg}}\right)} = 382.5\,\mathrm{m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.8(382.5\,\mathrm{m/s}) = 688.5\,\mathrm{m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.2332\,\mathrm{kg/m^3})[\pi(0.07\,\mathrm{m})^2/4](688.5\,\mathrm{m/s}) = 0.6179\,\mathrm{kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$Ma_1 = 1.8$: $T_1/T^* = 0.6089$, $T_{01}/T_0^* = 0.8363$

$Ma_2 = 1$: $T_2/T^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2 / T^*}{T_1 / T^*} = \frac{1}{0.6089} \quad \rightarrow \quad T_2 = T_1 / 0.6089 = (364.1\ \mathrm{K}) / 0.6089 = \mathbf{598\ K}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02} / T_0^*}{T_{01} / T_0^*} = \frac{1}{0.8363} \quad \rightarrow \quad T_{02} = T_{01} / 0.8363 = (600\ \mathrm{K}) / 0.8363 = 717.4\ \mathrm{K} \cong \mathbf{717\ K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\mathrm{air}} c_p (T_{02} - T_{01}) = (0.6179\ \mathrm{kg/s})(1.005\ \mathrm{kJ/kg \cdot K})(717.4 - 600)\ \mathrm{K} = \mathbf{72.9 kW}$$

Note that this is the highest temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature will cause the mass flow rate to decrease. Also, once the sonic conditions are reached, the thermodynamic temperature can be increased further by cooling the fluid and reducing the velocity (see the T-sdiagram for Rayleigh flow).