Previous Exam Problems

A piping system transports a fluid and supports a vertical load of 9 kN and a horizontal load of 13 kN (acting in the +x direction) at flange A. The pipe has an outside diameter of D = 200 mm and an inside diameter of d = 176 mm. Determine the principal stresses, the maximum shear Stress at points H and K.

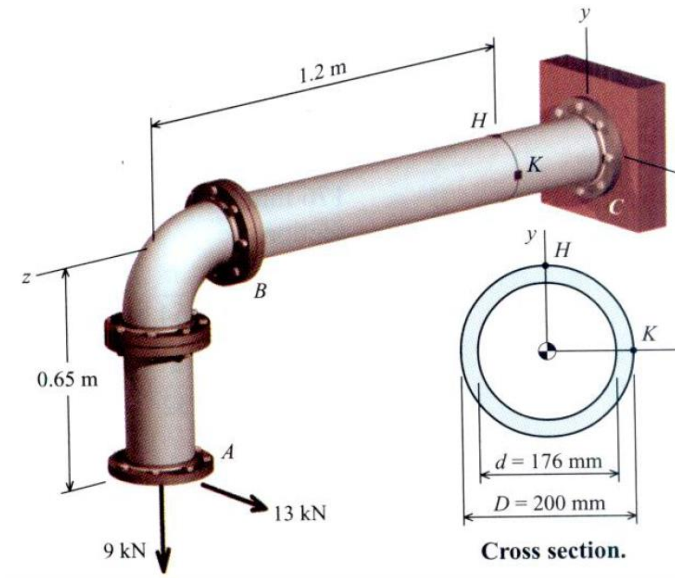
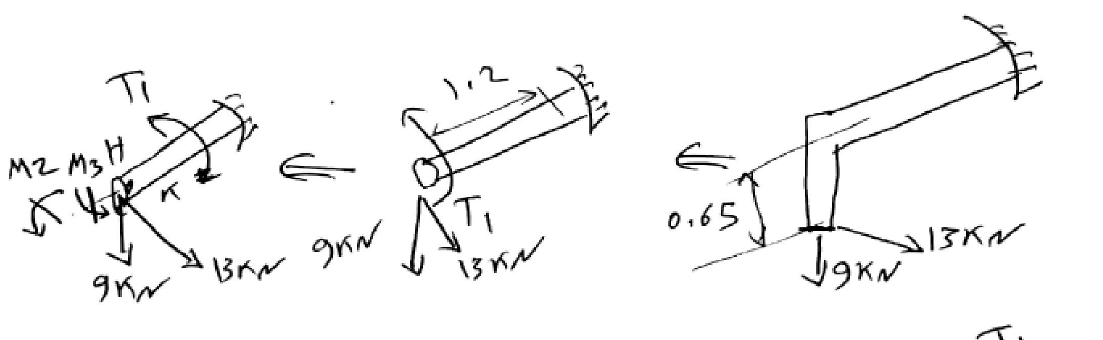


Figure 1.

Solution

Given: two forces 9KN, 13KN, $d_o = 200$, $d_i = 176mm$

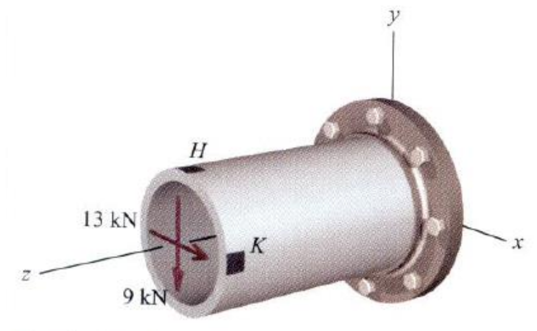
Req: $\sigma_{1,2}$, τ_{max} at H, K



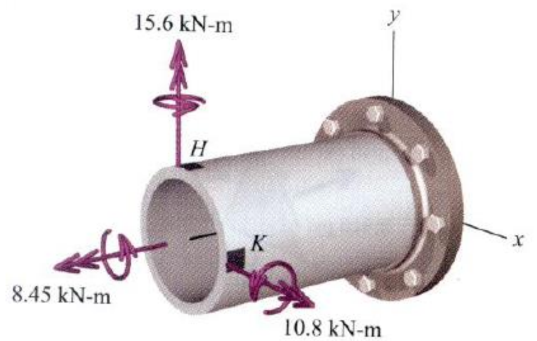
$T_1 = 13{,}000 \times 0.65 \times 1000 = 8.45 \times 10^6 \ N.mm$

$M_2 = 9 \times 1000 \times 1.2 \times 1000 = 10.8 \times 10^6 \ N.mm$

$M_3 = 13 \times 1000 \times 1.2 \times 1000 = 15.6 \times 10^6 \ N.mm$



Equivalent forces at the section that contains points *H* and *K*.



Equivalent moments at the section that contains points *H* and *K*.

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi}{64}\left(200^4 - 176^4\right) = 31.439 \times 10^6 \ mm^4 \quad (A)$$

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) = \frac{\pi}{32}\left(200^4 - 176^4\right) = 62.879 \times 10^6 \ mm^4 \quad (B)$$

shear due to T_1

$$\tau_{T_1} = \frac{T_1 r}{J} = \frac{8.45 \times 10^6 \times 200/2}{62.879 \times 10^6} \qquad \boxed{\mathrm{I}}$$

$$\tau_{T_1} = 13.438 \ MPa \quad (C)$$

Moment due to M2

$$\sigma_{M2} = \frac{M_2 y}{I} = \frac{10.8*10^6 * 200/2}{I} = 34.35 \text{ MPa}$$

Shear due to Moment M2 (force 9KN)

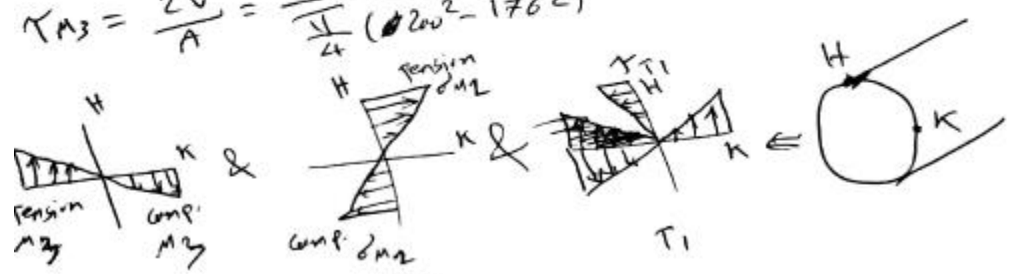
$$\tau_{M_2} = \frac{2V}{A} = \frac{2*9000}{\frac{\pi}{4}(d_o^2-d_i^2)} = \frac{2*9000}{\frac{\pi}{4}*(200^2-176^2)} = 2.539 \text{ MPa}$$

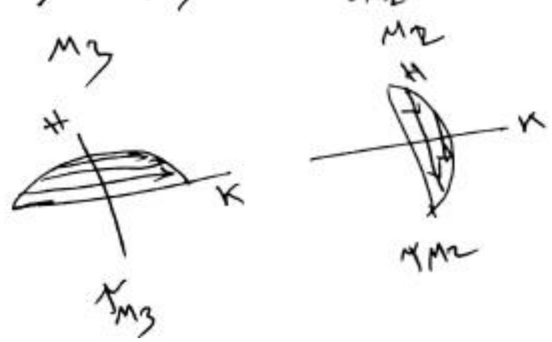
Moment due to M3

$$\sigma_{M_3} = \frac{M_3 y}{I} = \frac{15.6*10^6 * 200/2}{I} = 49.61 \text{ MPa}$$

Shear due to Moment M3 (force 13KN)

$$\tau_{M_3} = \frac{2V}{A} = \frac{2*13000}{\frac{\pi}{4}(200^2-176^2)} = 3.668 \text{ MPa}$$





for point K

$\sigma_K = -\sigma_{M_3} = -49.61\ MPa$

$\tau_K = \tau_{T_1} - \tau_{M_2} = 13.438 - 2.539 = 10.89\ MPa$

$\therefore \sigma_{x,K} = -49.61\ MPa \qquad \tau_{xy,K} = 10.89\ MPa \qquad \sigma_{y,K} = 0$

$$\sigma_{K\,\substack{max\\min}} = \frac{\sigma_{x,K}}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} = \frac{-49.61}{2} \pm \sqrt{\frac{(49.61)^2}{4} + (10.89)^2}$$

$= -24.8 \pm 27.09$

$\therefore \sigma_{max,K} = -51.89\ MPa \qquad \sigma_{min,K} = 2.285\ MPa$

$\tau_{max,K} = \sqrt{\frac{\sigma_{x,K}^2}{4} + \tau_{xy}^2} = 27.09\ MPa$

for point H

$\sigma_H = \sigma_{M_2} = 34.35\ MPa$

$\tau_H = \tau_{T_1} - \tau_{M_3} = 13.438 - 3.668 = 9.769\ MPa$

$\therefore \sigma_{x,H} = 34.35\ MPa \qquad \tau_{xy,H} = 9.769\ MPa \qquad \sigma_{y,H} = 0.0$

$$\therefore \sigma_{\substack{max\\min},H} = \frac{34.35}{2} \pm \sqrt{\frac{(34.35)^2}{4} + (9.769)^2}$$

$= 17.17 \pm 19.758$

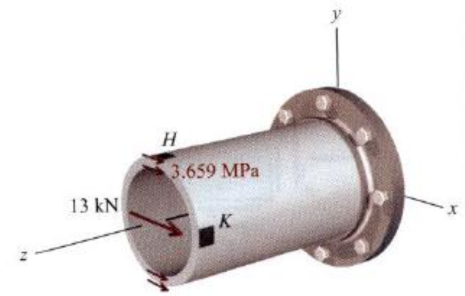
$\sigma_{max,H} = 36.93\ MPa \qquad \sigma_{min,H} = -2.583\ MPa$

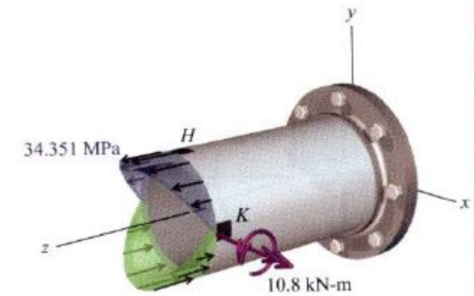
$\therefore \tau_{max,H} = \sqrt{\frac{(34.35)^2}{4} + (9.769)^2} = 19.758\ MPa$

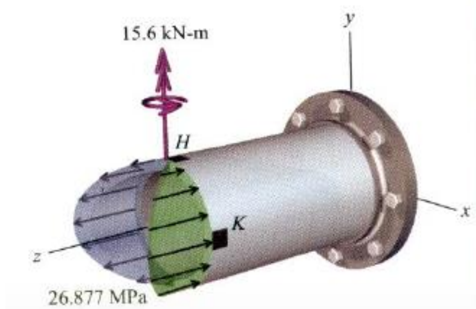
for K $\Rightarrow$ $\sigma_{max} = -51.89\ MPa$, $\sigma_{min} = 2.285\ MPa$, $\tau_{max} = 27.09\ MPa$

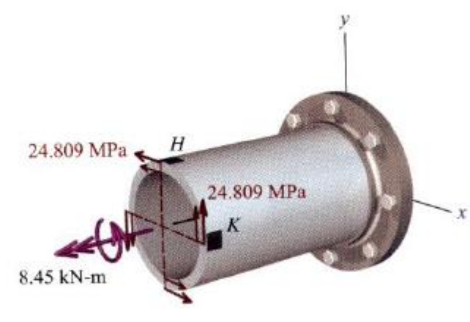
for H $\Rightarrow$ $\sigma_{max} = 36.93\ MPa$, $\sigma_{min} = -2.58\ MPa$, $\tau_{max} = 19.758\ MPa$

3


y
H
3.659 MPa
13 kN
K
z
x


y
34.351 MPa
H
K
z
x
10.8 kN-m


15.6 kN-m
y
H
K
z
x
26.877 MPa


y
24.809 MPa
H
24.809 MPa
K
x
8.45 kN-m

The drill is jammed in the wall and is subjected to the torque and force shown in Fig. 1. Determine the state of stress at points A and B on the cross section of drill bit, in back, at section a – a .

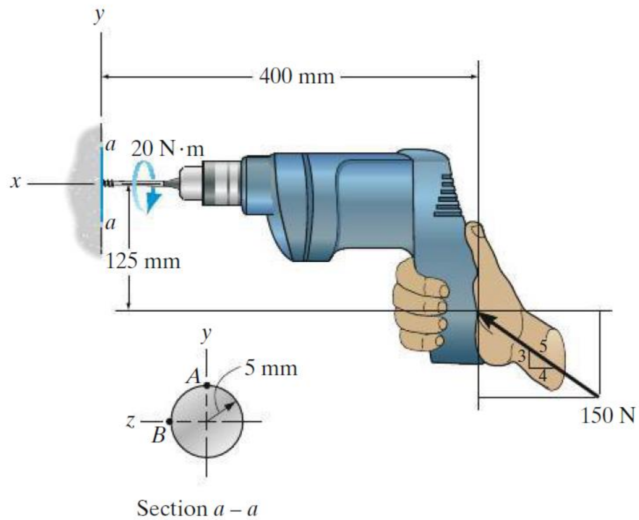


Figure 1.

Solution

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*,

$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad\qquad N = 120\text{ N}$

$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad\qquad V_y = 90\text{ N}$

$\Sigma M_x = 0; \; 20 - T = 0 \qquad\qquad T = 20\text{ N}\cdot\text{m}$

$$\Sigma M_z = 0; \; -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21\text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right)\text{m}^2$$

$$I_z = \frac{\pi}{4}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right)\text{m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right)\text{m}^4$$

Referring to Fig. *b*, Q_A is

$$Q_A = 0$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point A, $y = 0.005$ m. Then

$$\sigma_A = \frac{-120}{25\pi\left(10^{-6}\right)} - \frac{21(0.005)}{0.15625\pi\left(10^{-9}\right)} = -215.43\text{ MPa} = 215\text{ MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = \frac{V_y Q_A}{I_z t} = 0$$

The torsional shear stress developed at point A is

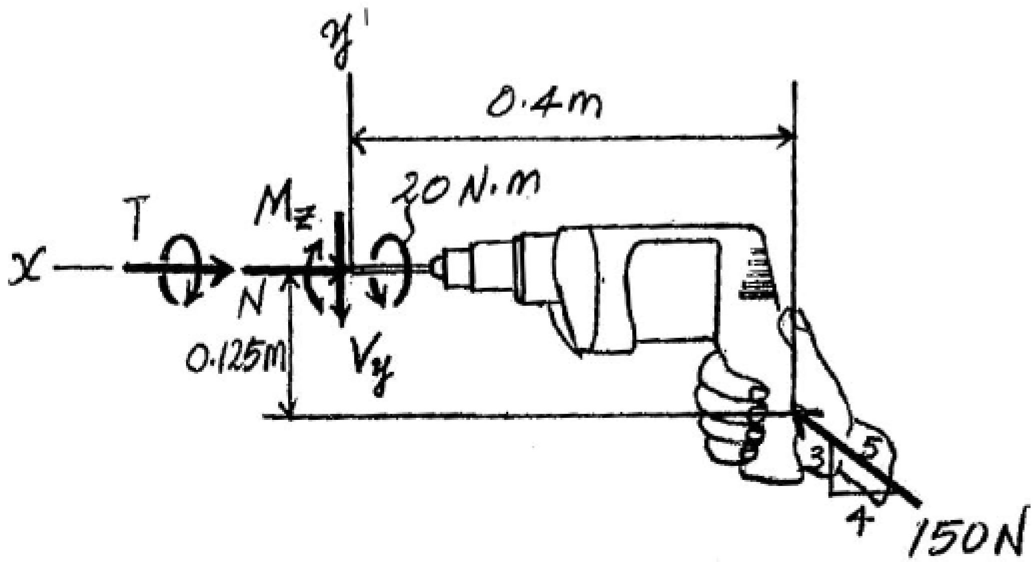
$$\left[(\tau_{xz})_T\right]_A = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

Thus,

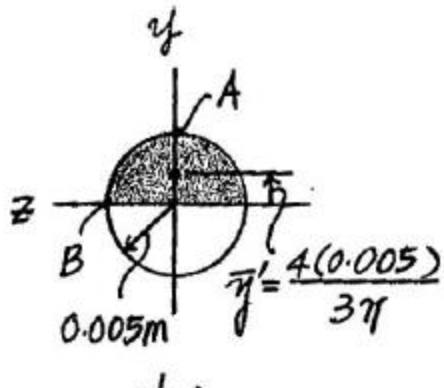
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A = 102 \text{ MPa} \qquad \textbf{Ans.}$$

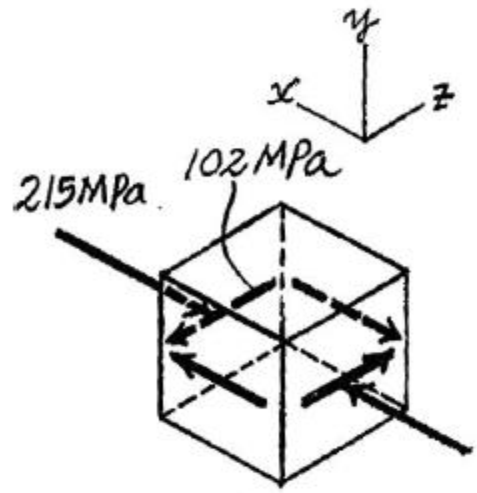
The state of stress at point A is represented on the element shown in Fig. c.



(a)



(b)



(c)

For Point B

Referring to Fig. *b*, Q_B is

$$Q_B = \overline{y}' A' = \frac{4(0.005)}{3\pi}\left[\frac{\pi}{2}\left(0.005^2\right)\right] = 83.333\left(10^{-9}\right)\mathrm{m}^3$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point B, $y = 0$. Then

$$\sigma_B = \frac{-120}{25\pi\left(10^{-6}\right)} - 0 = -1.528 \text{ MPa} = 1.53 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The transverse shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y Q_B}{I_z t} = \frac{90\left[83.333\left(10^{-9}\right)\right]}{0.15625\pi\left(10^{-9}\right)(0.01)} = 1.528 \text{ MPa}$$

The torsional shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_T\right]_B = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

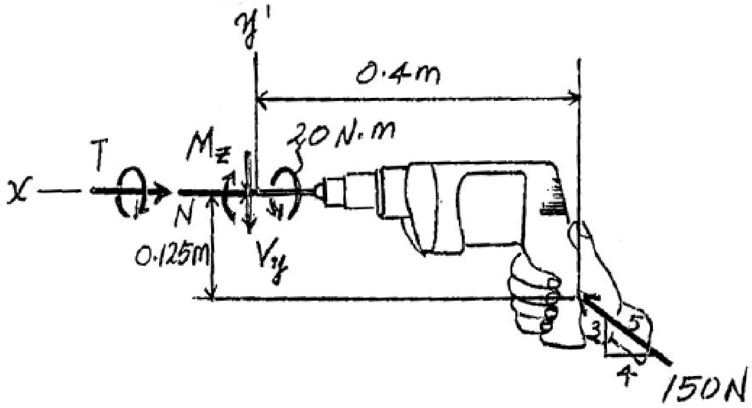
Thus,

$$\left(\tau_{xy}\right)_B = 0 \qquad \textbf{Ans.}$$

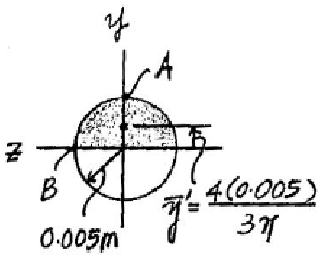
$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$

$$= 101.86 - 1.528 = 100.33 \text{ MPa} = 100 \text{ MPa} \qquad \textbf{Ans.}$$

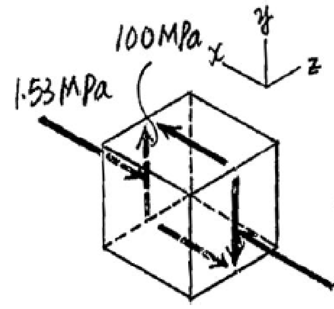
The state of stress at point B is represented on the element shown in Fig. *d*.



(a)



(b)



(d)

Ans:
$\sigma_B = 1.53$ MPa (C), $\tau_B = 100$ MPa