Stress Analysis

Lecture 2

ME 276

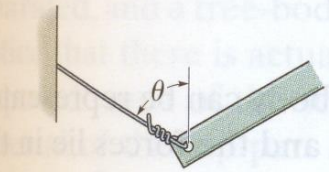
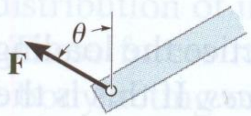
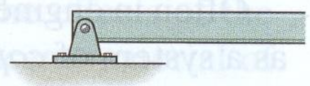
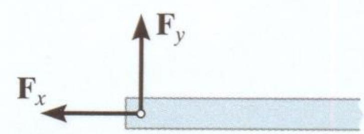
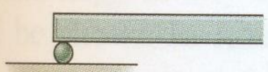
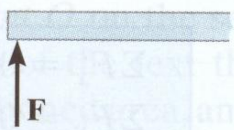
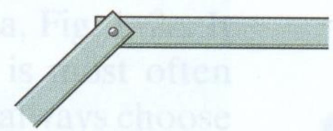
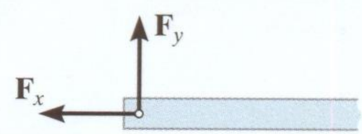
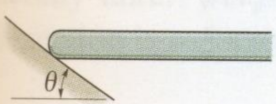
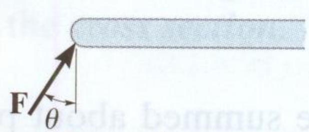
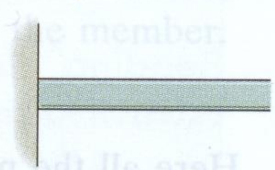
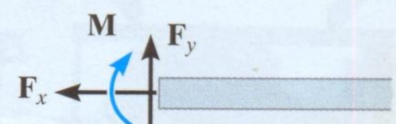
Spring 2017-2018

Dr./ Ahmed Mohamed Nagib Elmekawy

Review of Statics

Support Reactions

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

Review of Statics

Equations of Equilibrium

Equilibrium of a body requires both a *balance of forces* to prevent the body from translating or moving along a straight or curved path, and a *balance of moments*, to prevent the body from rotating. These conditions can be expressed mathematically by the two vector equations

$$\sum F = 0$$

$$\sum M_o = 0$$

$$\Downarrow$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

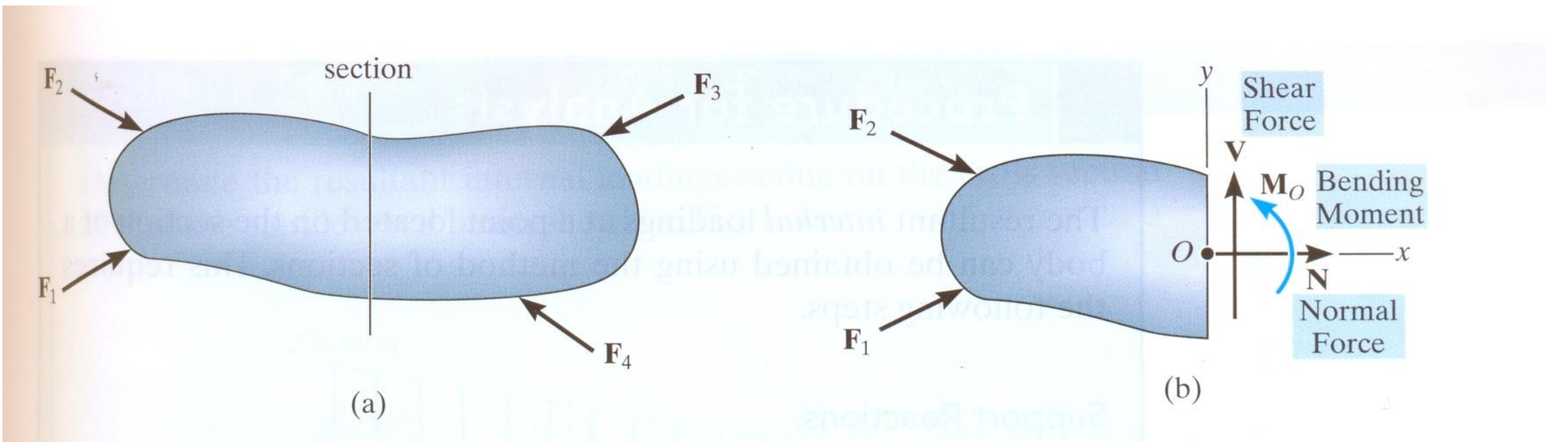
Review of Statics

Internal Loading

One of the most important applications of statics in the analysis of problems involving mechanics of materials is to be able to determine the resultant force and moment acting within a body, which are necessary to hold the body together when the body is subjected to external loads. For example consider the body shown in the figure which held in equilibrium by the four forces. In order to obtain the internal loadings acting on a specific region within the body, it is necessary to use the method of sections. This requires that an imaginary section or cut to be made through the region where the internal loadings are to be determined. The two parts of the body are then separated and a free-body diagram of one of the parts is drawn.

Review of Statics

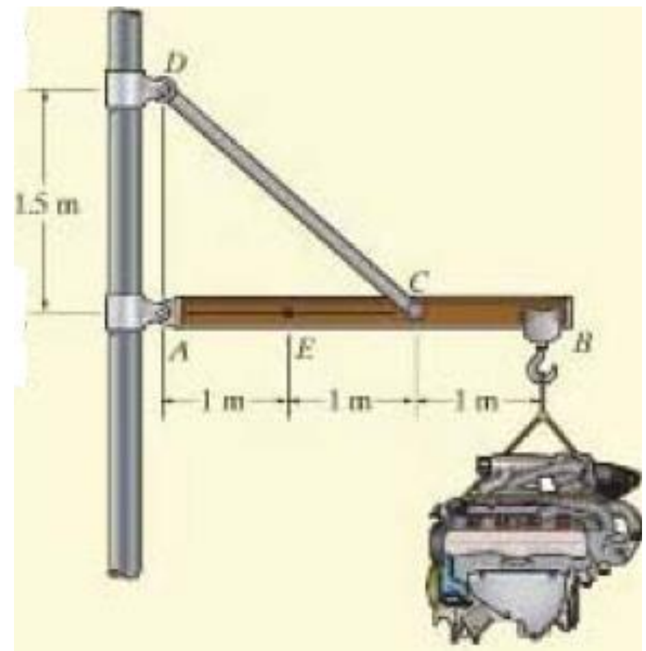
Internal Loading



Review of Statics

Example

The 500-kg engine is supported from the crane boom in the attached figure. Determine the resultant internal loading acting on the cross section of the boom at point E.



Support reaction

$$\sum M_A = 0 \qquad \Rightarrow F_{CD}\left(\frac{1.5}{2.5}\right)*2-(500*9.8*3)=0$$

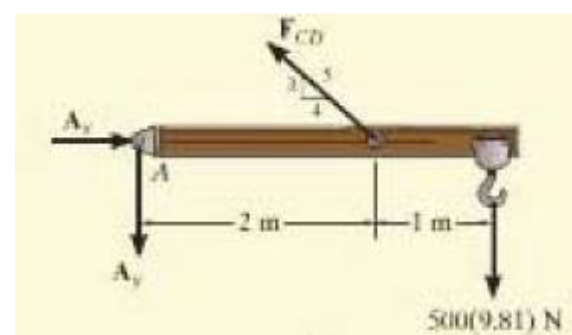
$$F_{CD} = 122625\,N$$

$$\sum F_x = 0 \qquad \Rightarrow A_x - 122625*\left(\frac{2}{2.5}\right)=0$$

$$A_x = 9810\,N$$

$$\sum F_y = 0 \qquad \Rightarrow -A_y + 122625*\left(\frac{1.5}{2.5}\right)-500*9.8=0$$

$$A_y = 2452.5\,N$$



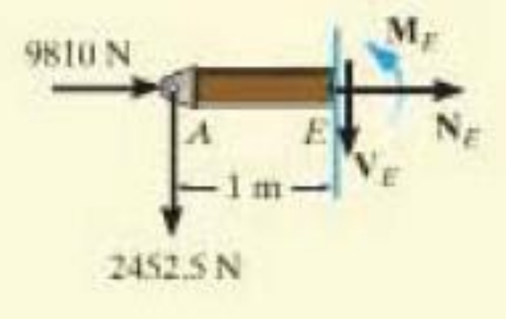
Review of Statics

$$\sum F_x = 0 \qquad \Rightarrow N_E + 9810 = 0$$

$$N_E = -9810\,N$$

$$\sum F_y = 0 \qquad \Rightarrow -V_E - 2452.5 = 0$$

$$V_E = -2452.5\,N$$



$$\sum M_E = 0 \qquad \Rightarrow M_E + (2452.5*1) = 0$$

$$M_E = -2452.5\,N.m$$

Axial Stress

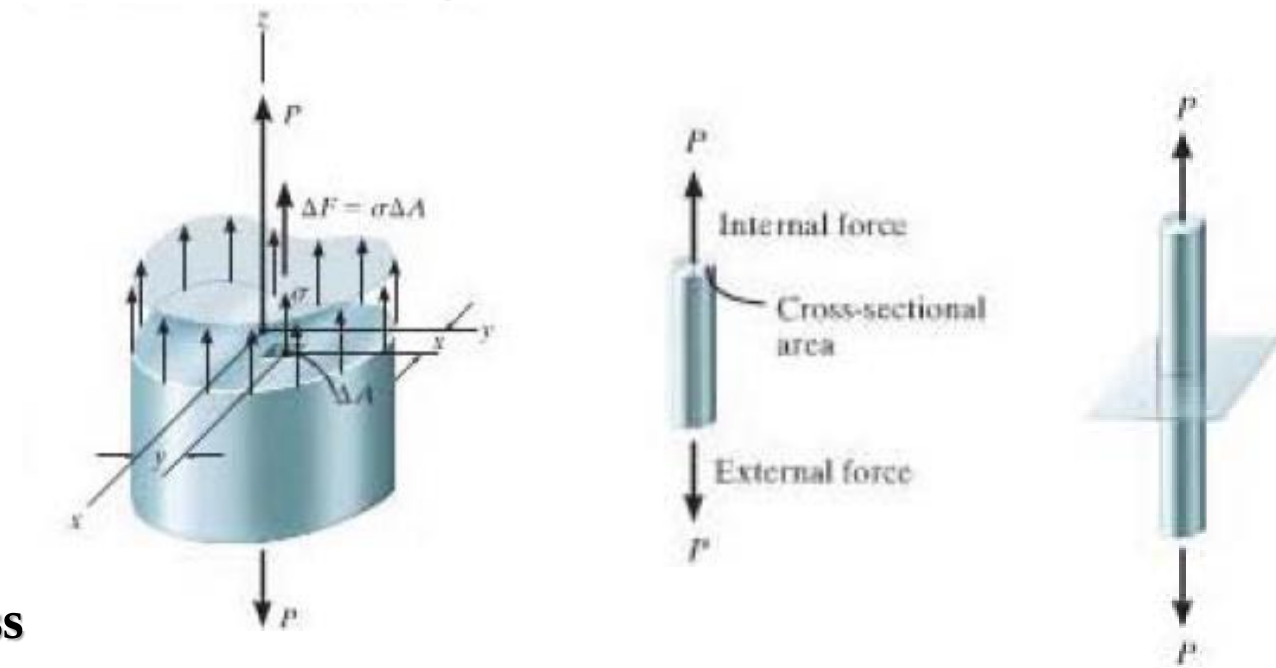
- The intensity of force, or force per unit area, acting normal to the area is defined as the normal stress, σ. If the normal force or stress pulls on the area it is referred to *tensile stress*, whereas if it pushes on the area it is called compressive stress.
- The intensity of force, or force per unit area, acting tangent to the area is called shear stress, τ.

Average Normal Stress in an Axially Loaded Bar

$$\sigma = \pm \frac{P}{A}$$

\+ for tensile stress

\- for compressive stress



Axial Stress

$$\sigma = \frac{P}{A} \qquad (1.5)$$

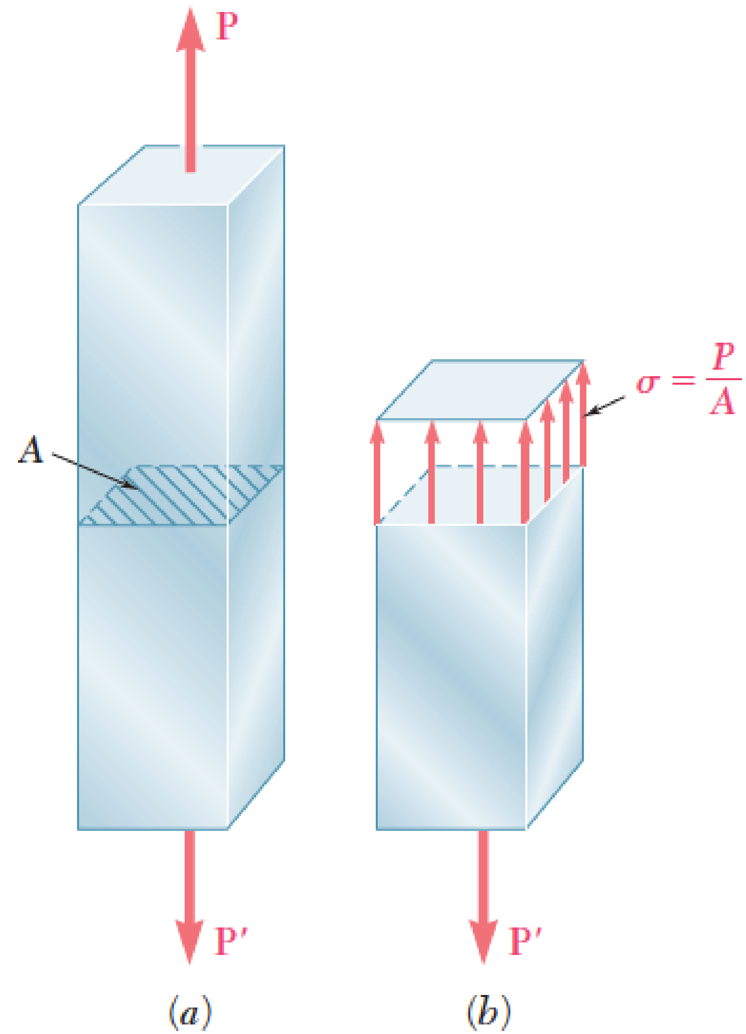


Fig. 1.8 (a) Member with an axial load. (b) Idealized uniform stress distribution at an arbitrary section.

A positive sign indicates a tensile stress (member in tension), and a negative sign indicates a compressive stress (member in compression).

Axial Stress

$$P = \int dF = \int_A \sigma \, dA$$

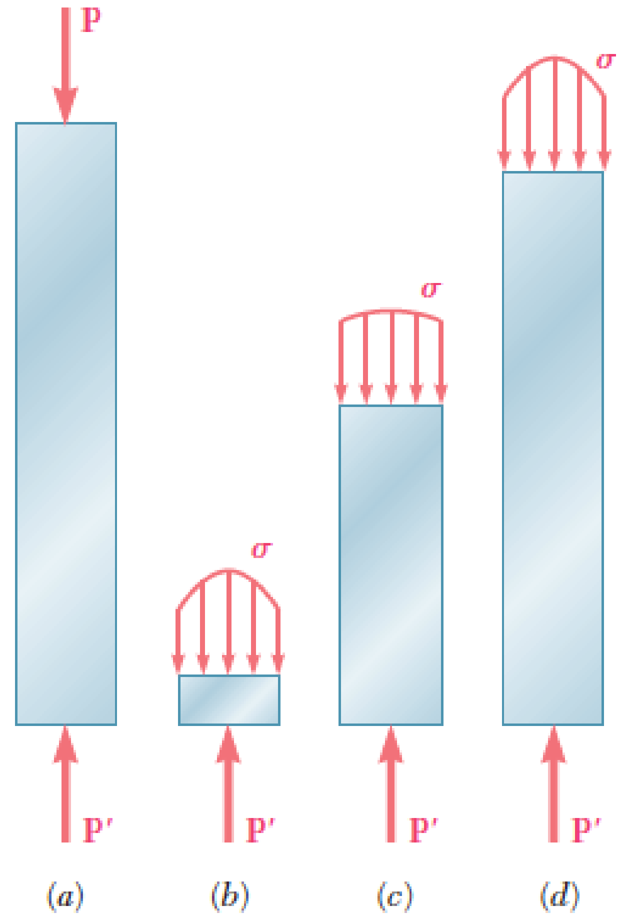


Fig. 1.10 Stress distributions at different sections along axially loaded member.

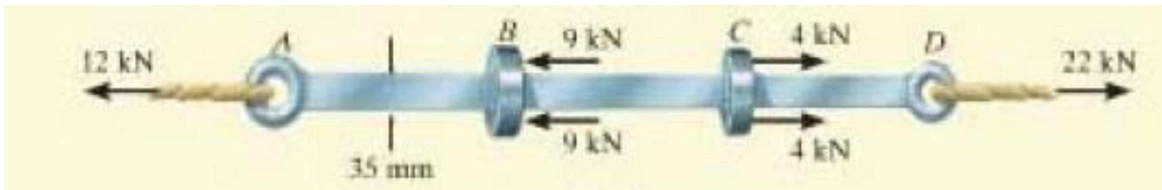
Tensile Test Video

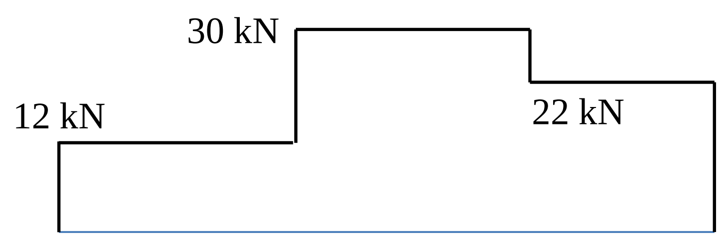
https://www.youtube.com/watch?v=D8U4G5kcpcM

Axial Stress

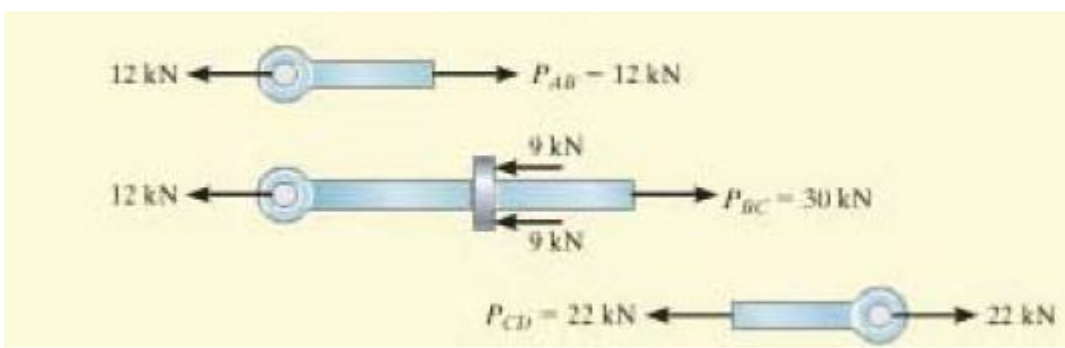
Example

The bar in the attached figure has a constant width of 35 mm and a thickness of 10 mm. Determine the largest normal stress in the bar when it is subjected to the loading shown.





Normal force diagram



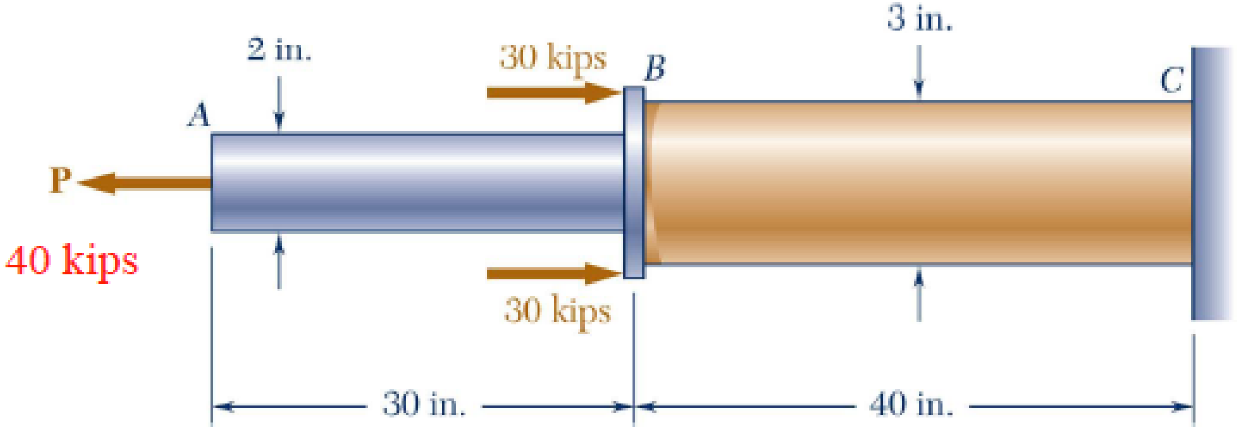
Axial Stress

The largest normal stress in the bar = σ_{BC}

$$\sigma_{BC} = \frac{30*1000}{35*10} = 85.7\,\text{MPa}$$

Example

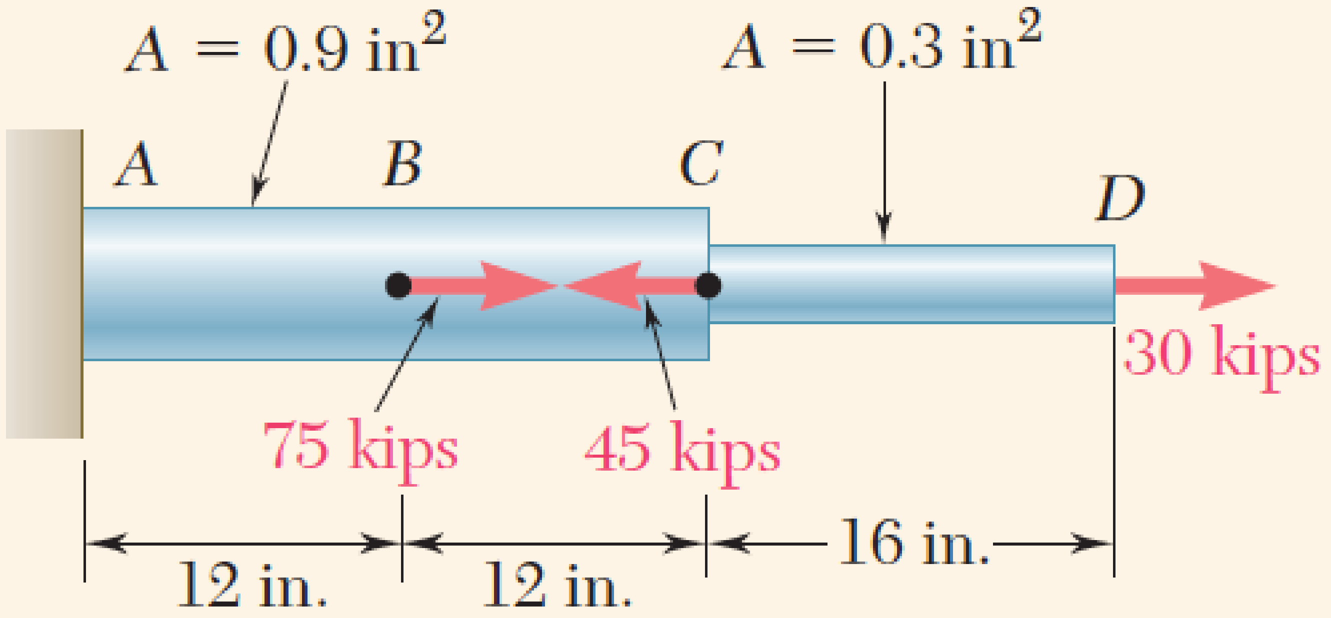
- Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the average normal stress at the midsection of (a) rod AB and (b) rod BC.



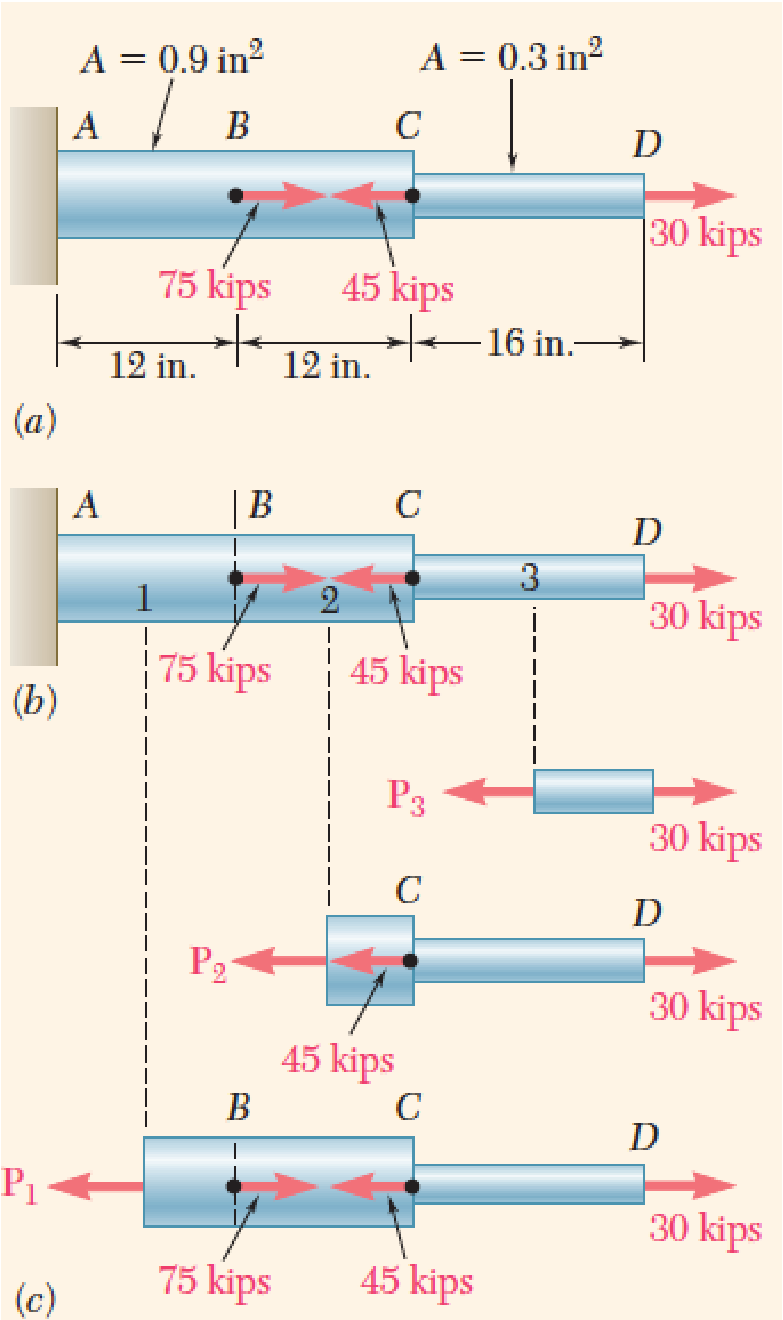
Example

Concept Application 2.1

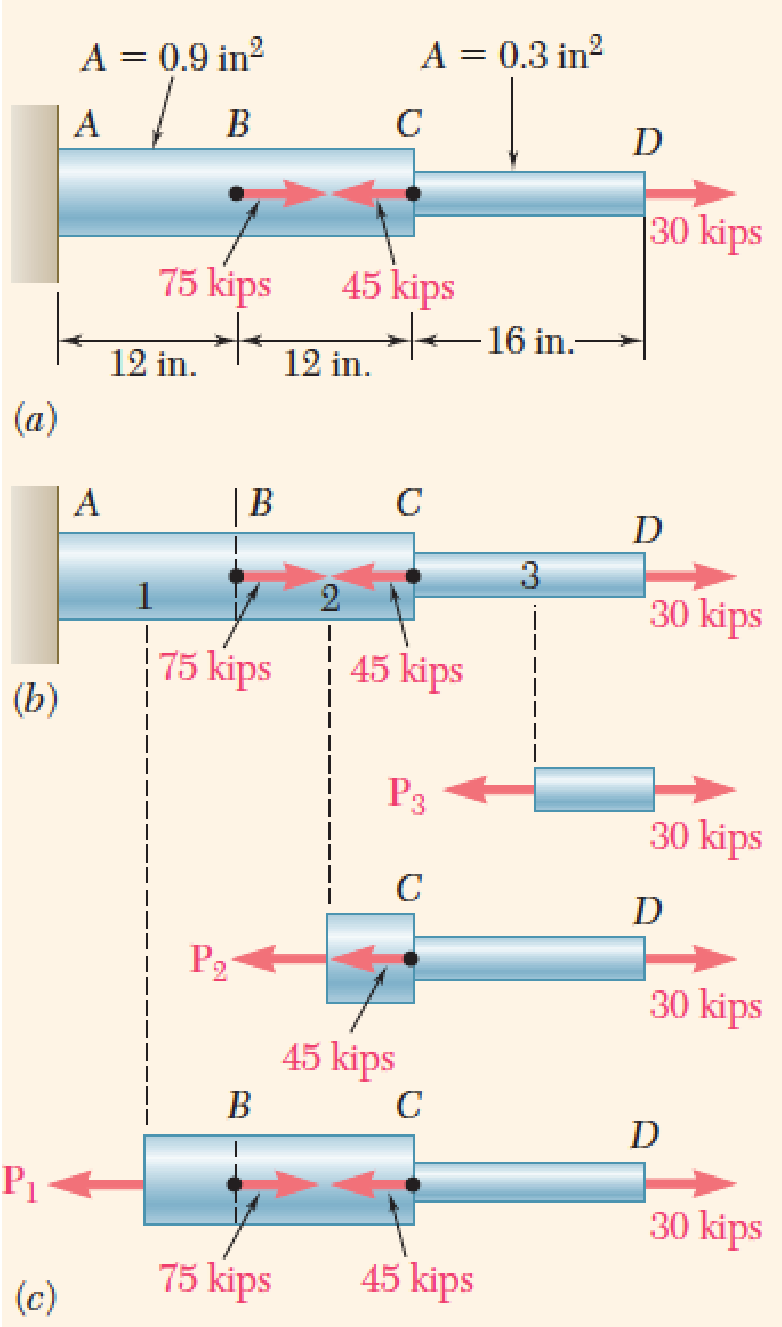
Determine the deformation of the steel rod shown in Fig. 2.19*a* under the given loads ($E = 29 \times 10^6$ psi).



(*a*)



A = 0.9 in²
A = 0.3 in²
A
B
C
D
30 kips
75 kips
45 kips
12 in.
12 in.
16 in.
(a)
A
B
C
D
1
2
3
30 kips
75 kips
45 kips
(b)
P3
30 kips
C
D
P2
30 kips
45 kips
B
C
D
P1
30 kips
75 kips
45 kips
(c)



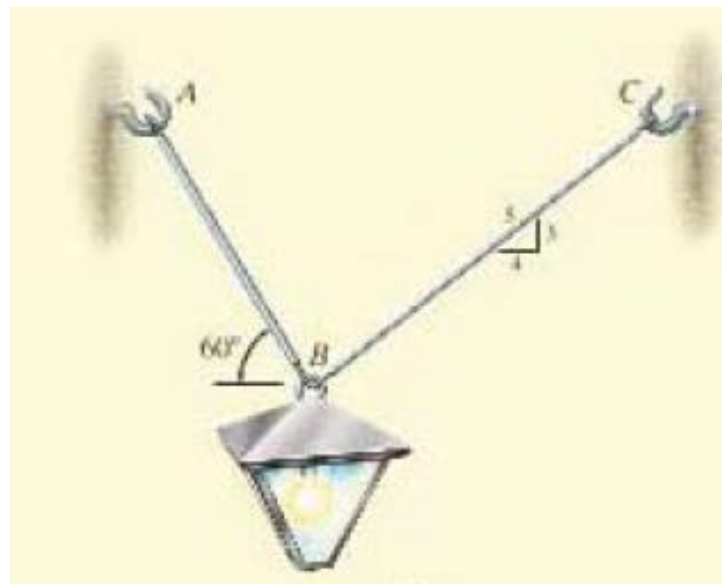
$$P_1 = 60 \text{ kips} = 60 \times 10^3 \text{ lb}$$
$$P_2 = -15 \text{ kips} = -15 \times 10^3 \text{ lb}$$
$$P_3 = 30 \text{ kips} = 30 \times 10^3 \text{ lb}$$

Using Eq. (2.10)

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$
$$= \frac{1}{29 \times 10^6}\left[\frac{(60 \times 10^3)(12)}{0.9} + \frac{(-15 \times 10^3)(12)}{0.9} + \frac{(30 \times 10^3)(16)}{0.3}\right]$$
$$\delta = \frac{2.20 \times 10^6}{29 \times 10^6} = 75.9 \times 10^{-3} \text{ in.}$$

Example

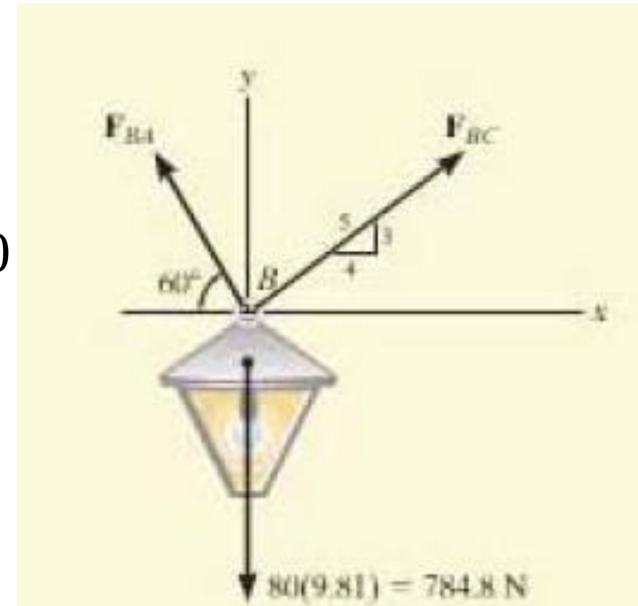
The 80-kg lamp is supported by two rods *AB* and *BC* as shown in the attached figure. If *AB* has a diameter of 10 mm and *BC* has a diameter of 8 mm. Determine which rod is subjected to greater normal stress.



$$\sum F_x = 0 \quad \Rightarrow \quad F_{BC}\left(\frac{4}{5}\right) - F_{BA}\cos 60 = 0$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{BC}\left(\frac{3}{5}\right) + F_{BA}\sin 60 - 784.8 = 0$$

$$F_{BC} = 395.2\,N \qquad F_{BA} = 632.4\,N$$



$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{F_{BC}}{\frac{\pi}{4}d_{BC}^2} = \frac{395.2}{\frac{\pi}{4}(8)^2} = 7.86\,\mathrm{MPa}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{F_{BA}}{\frac{\pi}{4}d_{BA}^2} = \frac{632.4}{\frac{\pi}{4}(10)^2} = 8.05\,\mathrm{MPa}$$

The rod *AB* is subjected to the greater normal stress

Axial Stress

Allowable Stress

- An engineer on charge of the design of a structural or mechanical element must restrict the stress in the material to a level that will be safe.
- So it becomes necessary to perform the calculations using a safe or allowable stress.
- To ensure safety, it is necessary to choose an allowable stress that restrict the applied load to one that is less than the load the member can fully support.
- One method of specifying the allowable load for the design or analysis of a member is to use a number called the *factor of safety*.

$$f.s. = \frac{\sigma_{fail}}{\sigma_{allow}} \qquad f.s. = \frac{\tau_{fail}}{\tau_{allow}}$$

Strain

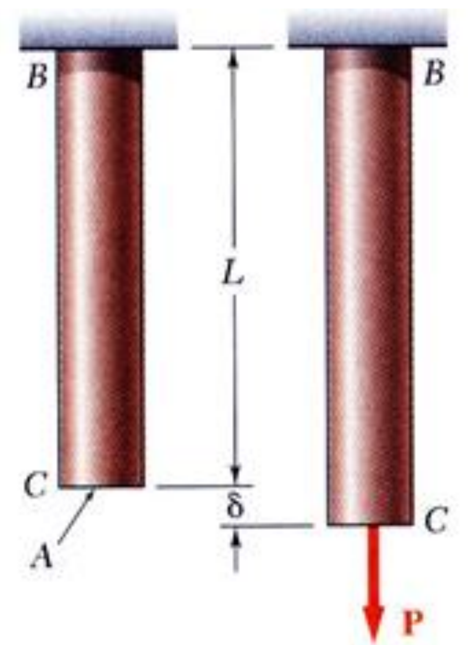
Deformation

Whenever a force is applied to a body, it will tend to change the body's shape and size. These changes are referred to as ***deformation***, and they may be either highly visible or practically unnoticeable.

Normal Strain

The normal strain ε is given by

$$\varepsilon = \frac{\Delta L}{L_o} = \frac{\delta}{L_o}$$



ΔL **is the deformation under axial load, namely, the change in length of the member**

L_o **is the original length**

ε **is dimensionless**

Strain

Stress-Strain/Modulus of Elasticity

The normal stress is related to the normal strain by

$$\sigma = E\varepsilon$$

E is the modulus of elasticity (also called Young's Modulus) of the material of the rod.

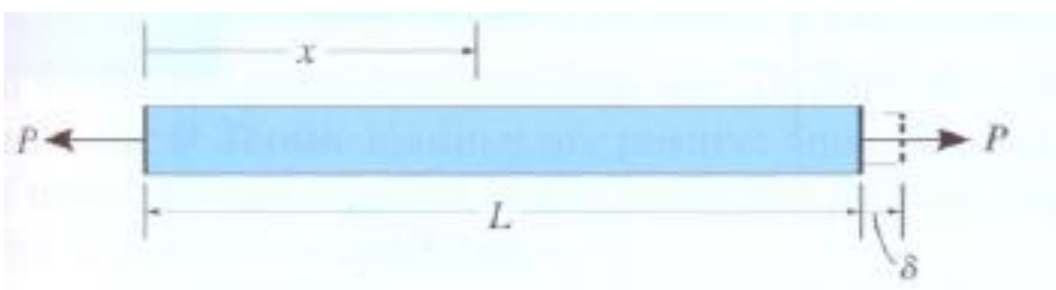
Elastic Deformation of an Axially Loaded Member

Constant Load and Cross-sectional Area

$$\delta = \frac{PL}{AE}$$

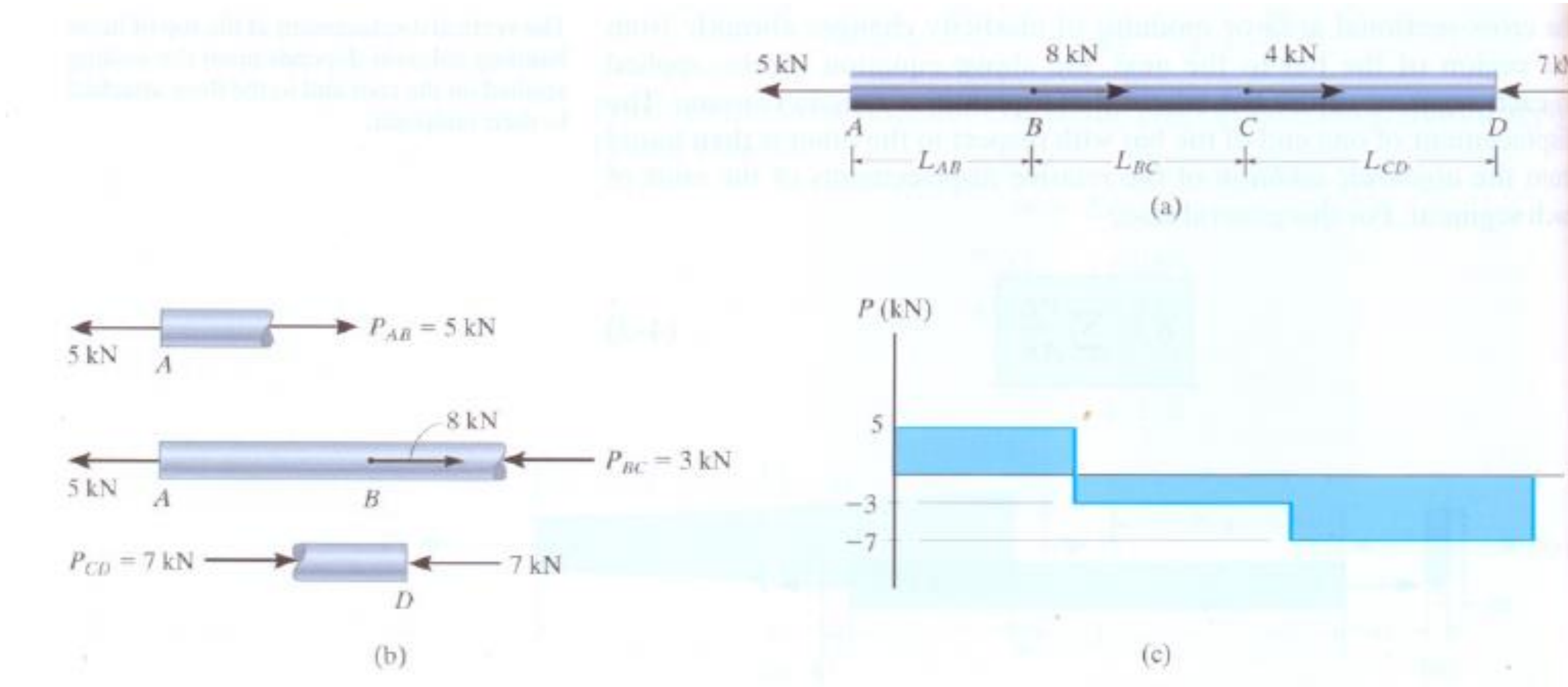
If the bar is subjected to several different axial forces along its length, or the cross-sectional area or modulus of elasticity changes abruptly from one region of the bar to the next, the above equation can be applied to each segment of the bar.

$$\delta = \sum \frac{PL}{AE}$$



Elastic Deformation of an Axially Loaded Member

Sign Convention



$$\delta_{A/D} = \sum \frac{PL}{AE} = \frac{(5\,kN)L_{AB}}{AE} + \frac{(-3\,kN)L_{BC}}{AE} + \frac{(-7\,kN)L_{CD}}{AE}$$

Elastic Deformation of an Axially Loaded Member

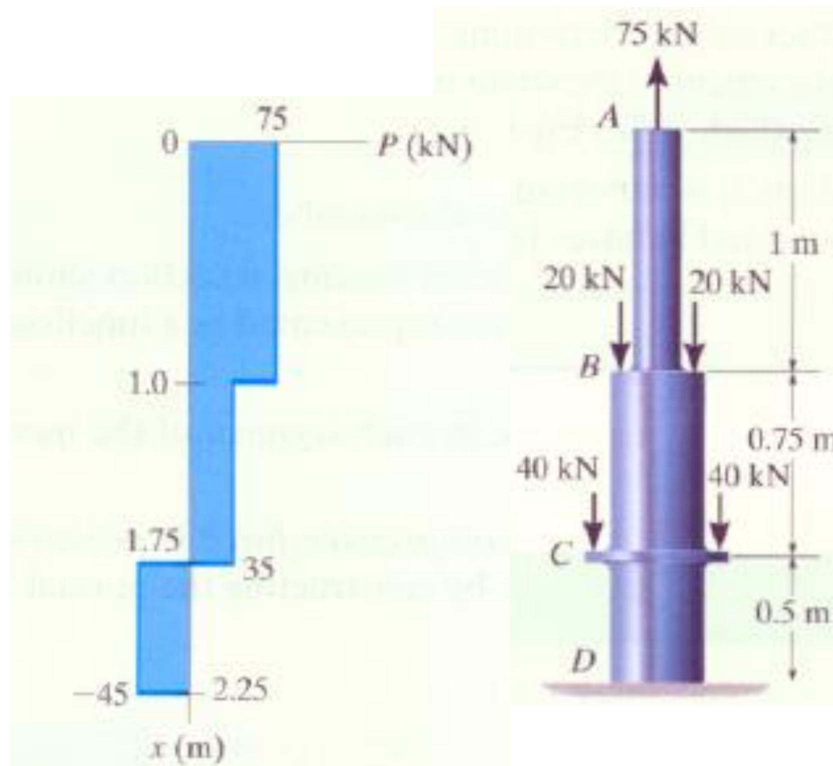
Example

The A-36 steel bar shown in the figure is made from two segments having cross-sectional areas of A_{AB} = 600 mm² and A_{BD} = 1200 mm². Determine the vertical displacement of end *A* and the displacement of *B* relative to *C*.

$$\delta_A = \sum \frac{PL}{AE}$$

$$= \frac{75000*1000}{600*200*10^3} + \frac{35000*750}{1200*200*10^3} - \frac{45000*500}{1200*200*10^3}$$

$$= 0.641 \text{ mm}$$

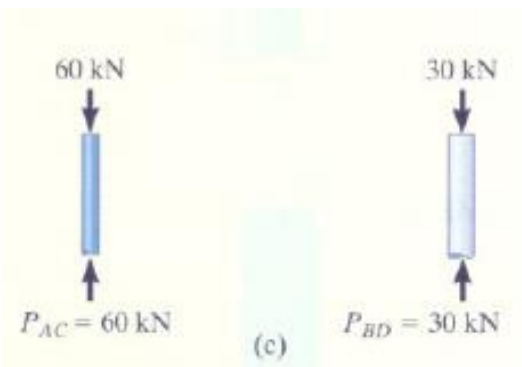


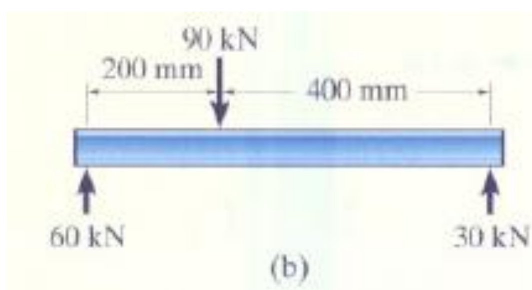
Elastic Deformation of an Axially Loaded Member

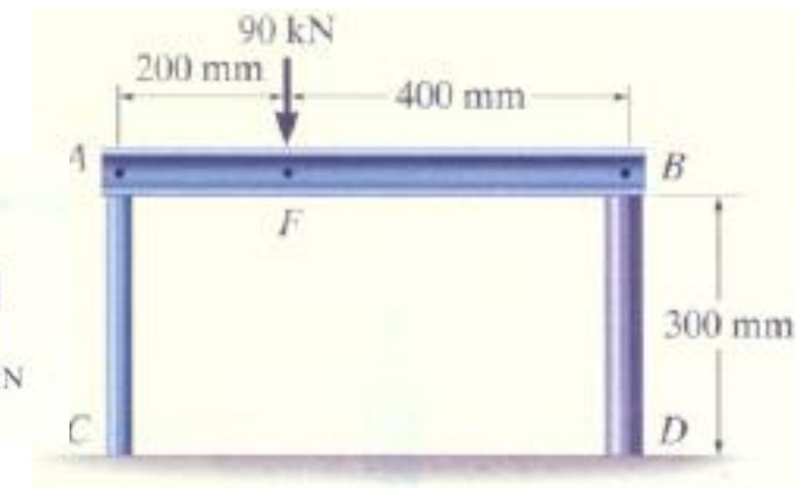
$$\delta_{B/C} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{35000 * 750}{1200 * 200 * 10^3} = 0.109 \text{ mm}$$

Example

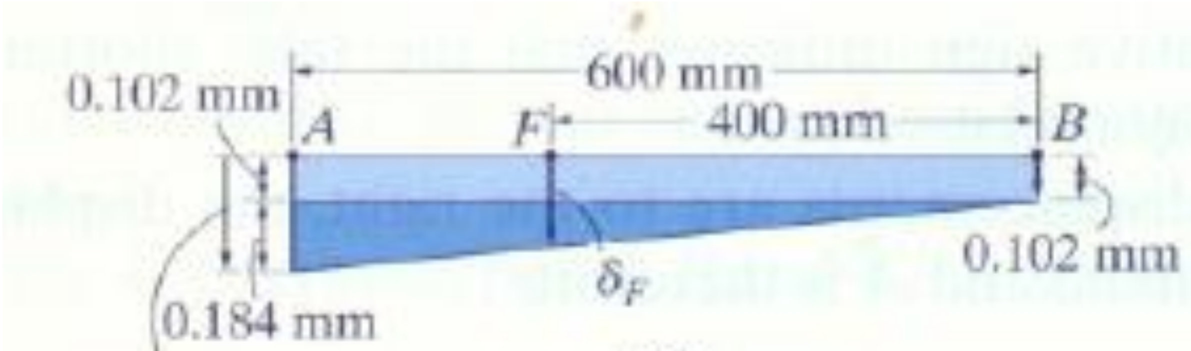
Rigid beam *AB* rests on the two short posts shown in the attached figure. *AC* is made of steel and has a diameter of 20 mm, and *BD* is made of aluminum and has a diameter of 40 mm. Determine the displacement of point *F* on *AB* if a vertical load of 90 kN is placed over this point . Take E_{st} = 200 Gpa, E_{al} = 70 Gpa







Elastic Deformation of an Axially Loaded Member



(d)

$$\delta_A = \frac{P_{AC} L_{AC}}{A_{AC} E_{st}} = \frac{-60000 * 300}{\pi (10)^2 * 200 * 10^3} = -0.286 \text{ mm}$$

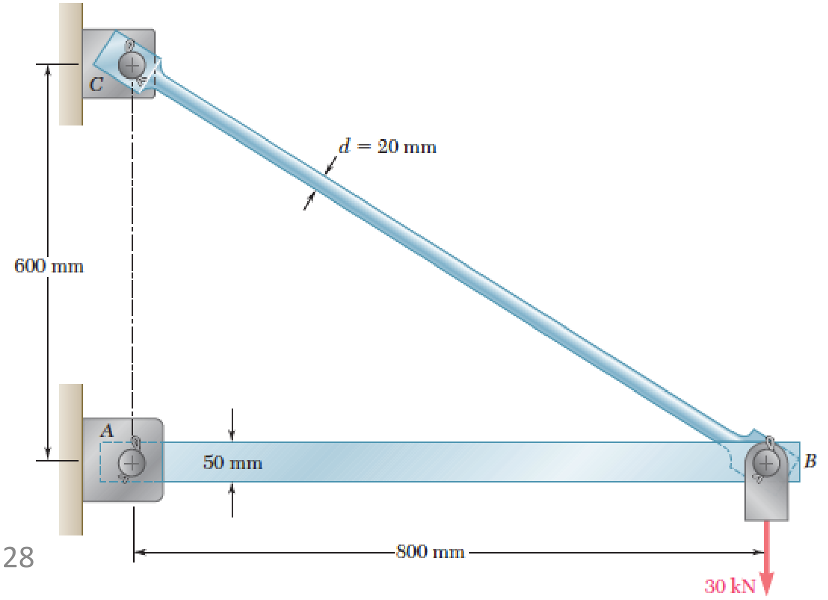
$$\delta_B = \frac{P_{BD} L_{BD}}{A_{BD} E_{al}} = \frac{-30000 * 300}{\pi (20)^2 * 70 * 10^3} = -0.102 \text{ mm}$$

$$\delta_F = 0.102 + 0.184 \left(\frac{400}{600} \right) = 0.225 \text{ mm}$$

Example

Concept Application 1.1

Considering the structure of Fig. 1.1 on page 5, assume that rod *BC* is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod *BC* safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was 50 kN. Recalling that the diameter of the rod is 20 mm, use Eq. (1.5) to determine the stress created in the rod by the given loading.



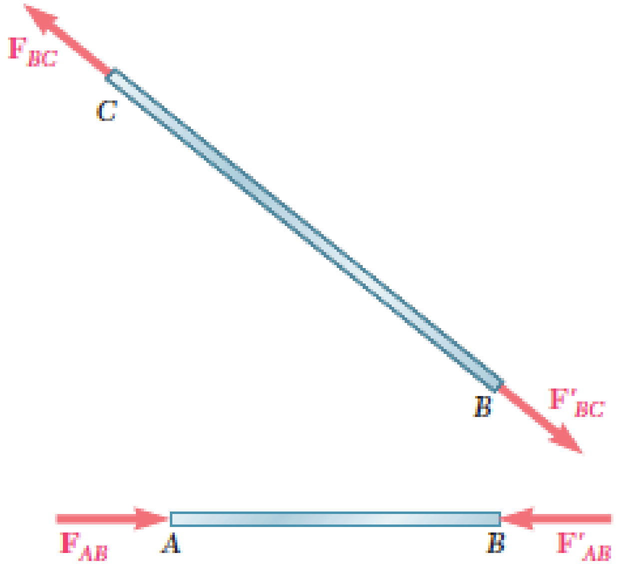


Fig. 1.5 Free-body diagrams of two-force members *AB* and *BC*.

$$P = F_{BC} = +50\ \text{kN} = +50 \times 10^3\ \text{N}$$

$$A = \pi r^2 = \pi\left(\frac{20\ \text{mm}}{2}\right)^2 = \pi(10 \times 10^{-3}\ \text{m})^2 = 314 \times 10^{-6}\ \text{m}^2$$

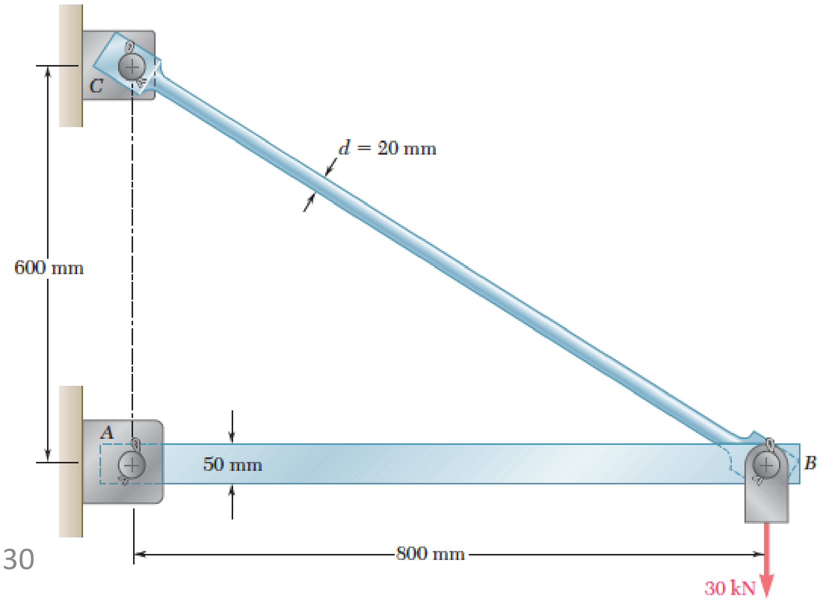
$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3\ \text{N}}{314 \times 10^{-6}\ \text{m}^2} = +159 \times 10^6\ \text{Pa} = +159\ \text{MPa}$$

Since σ is smaller than σ_{all} of the allowable stress in the steel used, rod *BC* can safely support the load.

Example

Concept Application 1.2

As an example of design, let us return to the structure of Fig. 1.1 on page 5 and assume that aluminum with an allowable stress $\sigma_{all} =$ 100 MPa is to be used. Since the force in rod BC is still $P = F_{BC} = 50$ kN under the given loading, from Eq. (1.5), we have



Concept Application 1.2

As an example of design, let us return to the structure of Fig. 1.1 on page 5 and assume that aluminum with an allowable stress σ_{all} = 100 MPa is to be used. Since the force in rod BC is still $P = F_{BC} = 50$ kN under the given loading, from Eq. (1.5), we have

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

and since $A = \pi r^2$,

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6} \text{ m}^2}{\pi}} = 12.62 \times 10^{-3} \text{ m} = 12.62 \text{ mm}$$

$$d = 2r = 25.2 \text{ mm}$$

Therefore, an aluminum rod 26 mm or more in diameter will be adequate.