



Stress Analysis

Lecture 3

ME 276

Spring 2017-2018

Dr./ Ahmed Mohamed Nagib Elmekawy

Axial Stress

$$\sigma = \frac{P}{A}$$

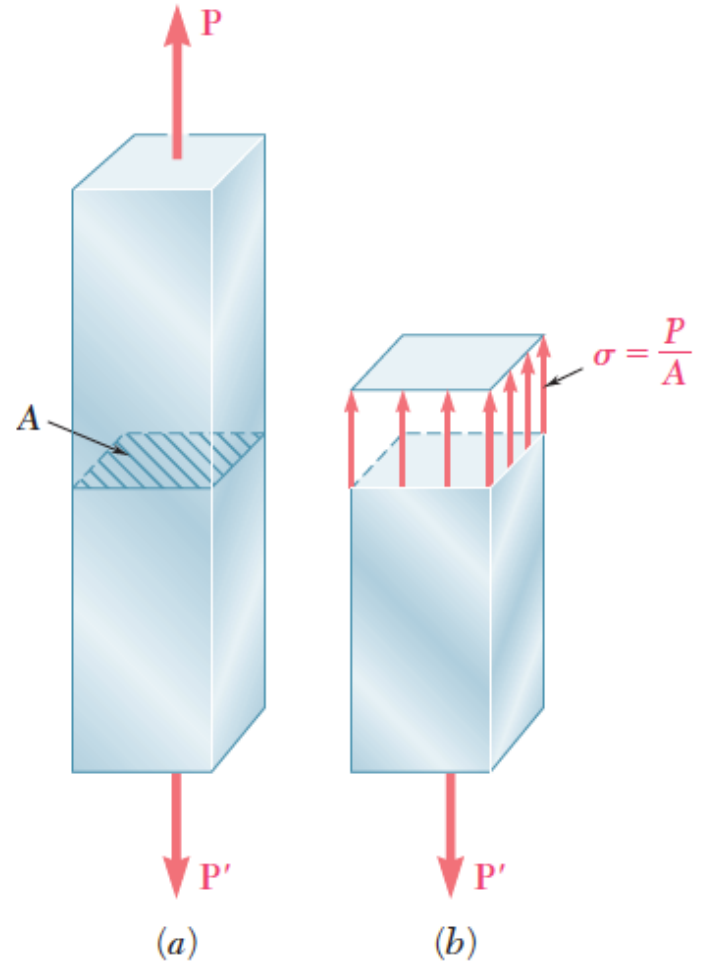
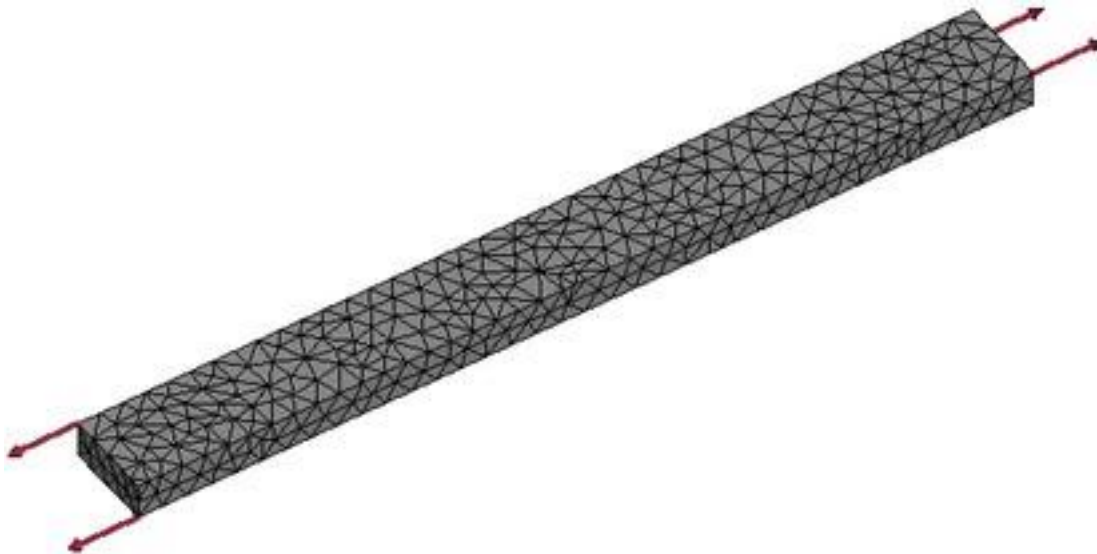
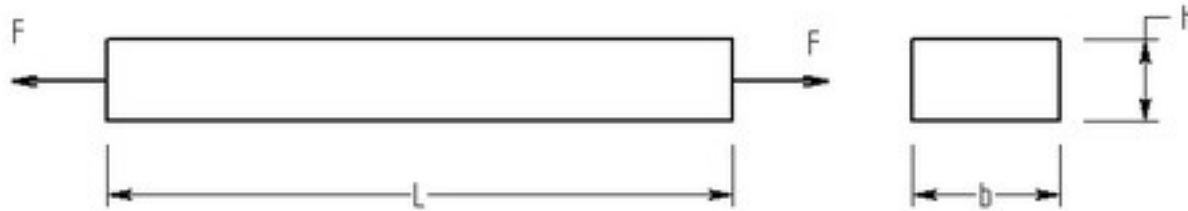
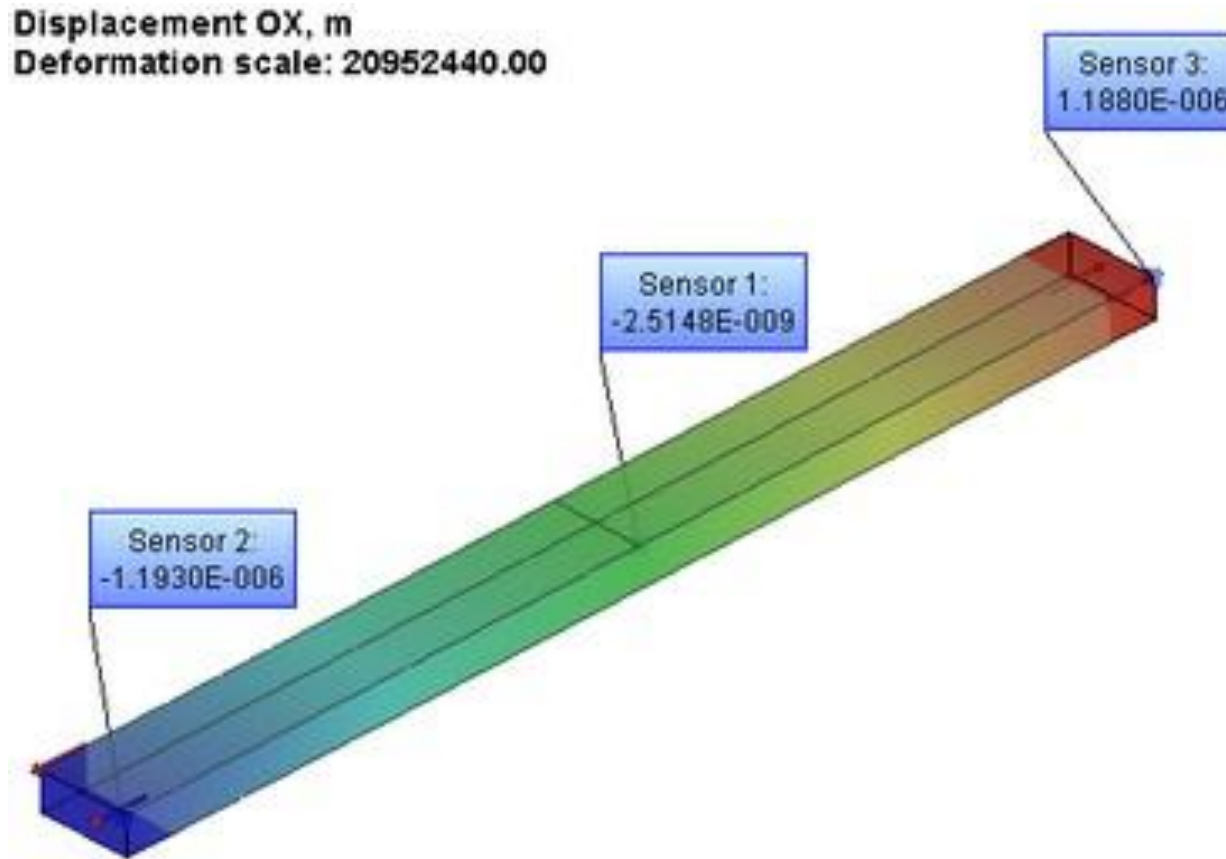


Fig. 1.8 (a) Member with an axial load.
(b) Idealized uniform stress distribution at an arbitrary section.

Beam under the action of two tensile forces

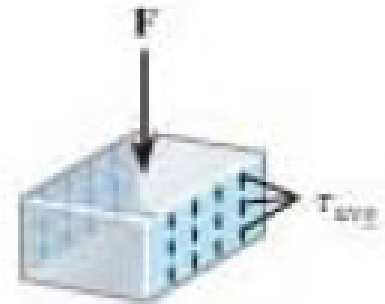
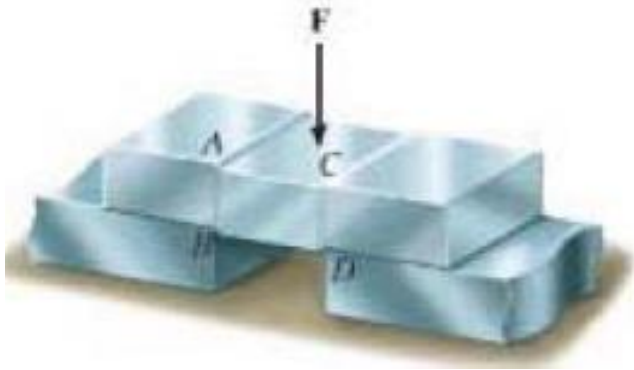


Beam under the action of two tensile forces



Shear Stress

Average Shear Stress



$$\tau = \frac{V}{A}$$

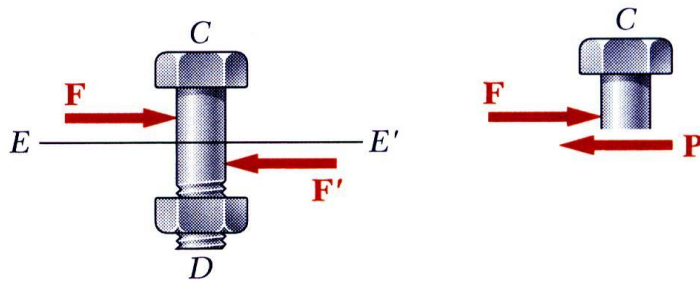
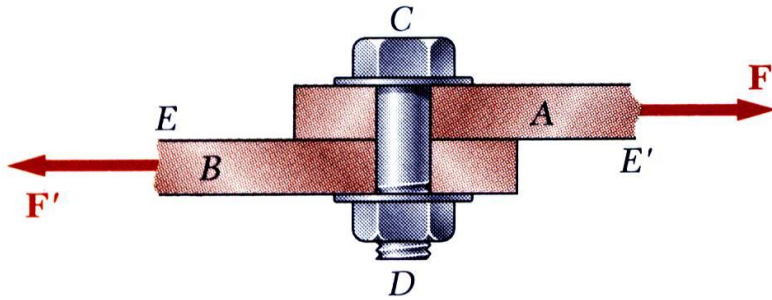
τ is the average shear stress at the section

V is internal resultant shear force at the section determined from the equations of equilibrium

A is the area at the section

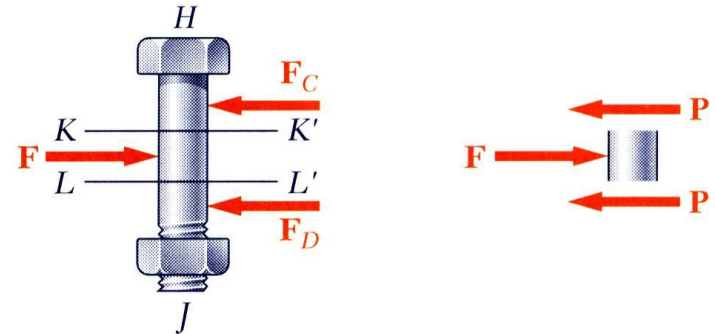
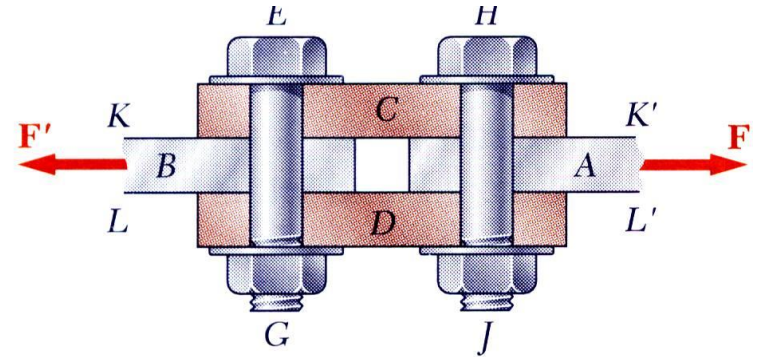
Stress

Single Shear



$$\tau = \frac{P}{A} = \frac{F}{A}$$

Double Shear

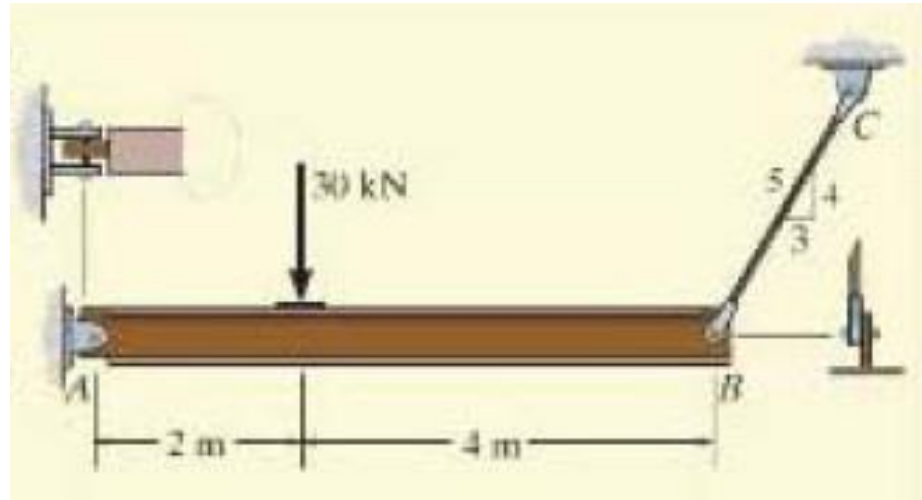
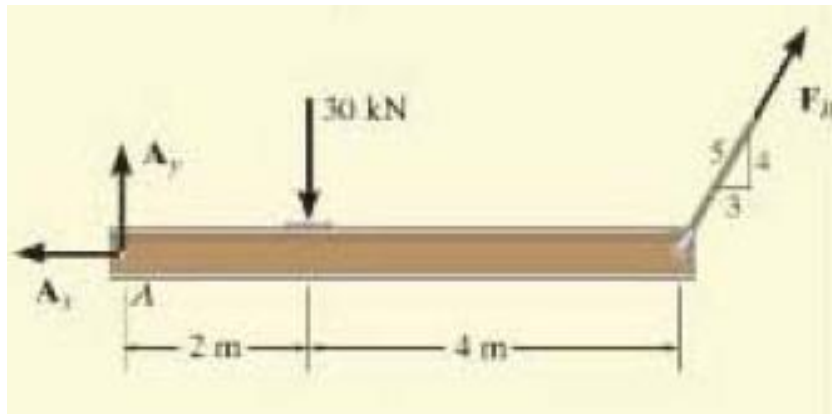


$$\tau = \frac{F}{2A}$$

Stress

Example 1

Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm diameter pin at B that support the beam in the attached figure.



$$\sum M_A = 0 \quad \Rightarrow \quad F_B \left(\frac{4}{5} \right) 6 - 30 * 2 = 0 \quad \Rightarrow \quad F_B = 12.5 \text{ kN}$$

$$\sum F_x = 0 \quad \Rightarrow \quad 12.5 \left(\frac{3}{5} \right) - A_x = 0 \quad \Rightarrow \quad A_x = 7.5 \text{ kN}$$

$$\sum F_y = 0 \quad \Rightarrow \quad 12.5 \left(\frac{4}{5} \right) + A_y - 30 = 0 \quad \Rightarrow \quad A_y = 20 \text{ kN}$$

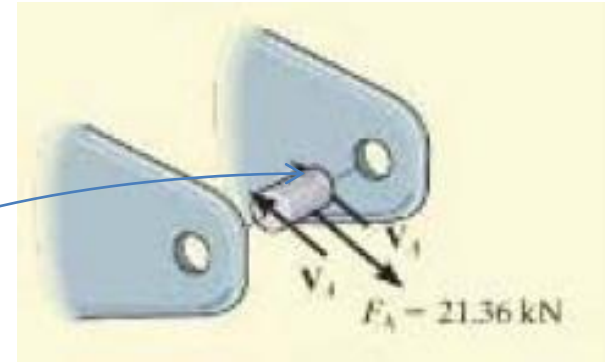
Stress

The resultant force acting on pin *A* is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.5)^2 + (20)^2} = 21.36 \text{ kN}$$

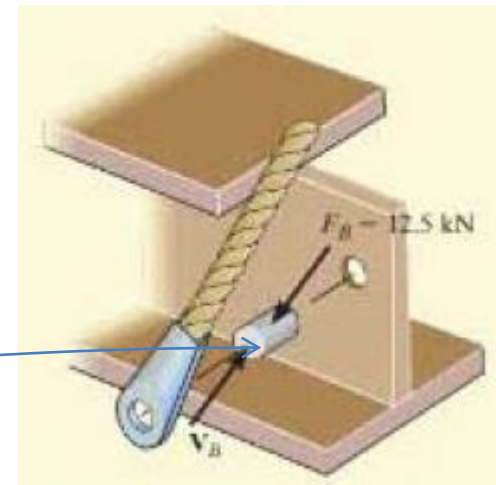
$$\tau_A = \frac{F_A}{2A_A} = \frac{21.36 * 1000}{2 * \frac{\pi}{4} (20)^2} = 34 \text{ MPa}$$

double shear



$$\tau_B = \frac{F_B}{A_B} = \frac{21.5 * 1000}{\frac{\pi}{4} (30)^2} = 17.7 \text{ MPa}$$

single shear



Allowable Stress

- An engineer on charge of the design of a structural or mechanical element must restrict the stress in the material to a level that will be safe.
- So it becomes necessary to perform the calculations using a safe or allowable stress.
- To ensure safety, it is necessary to choose an allowable stress that restrict the applied load to one that is less than the load the member can fully support.
- One method of specifying the allowable load for the design or analysis of a member is to use a number called the *factor of safety*.

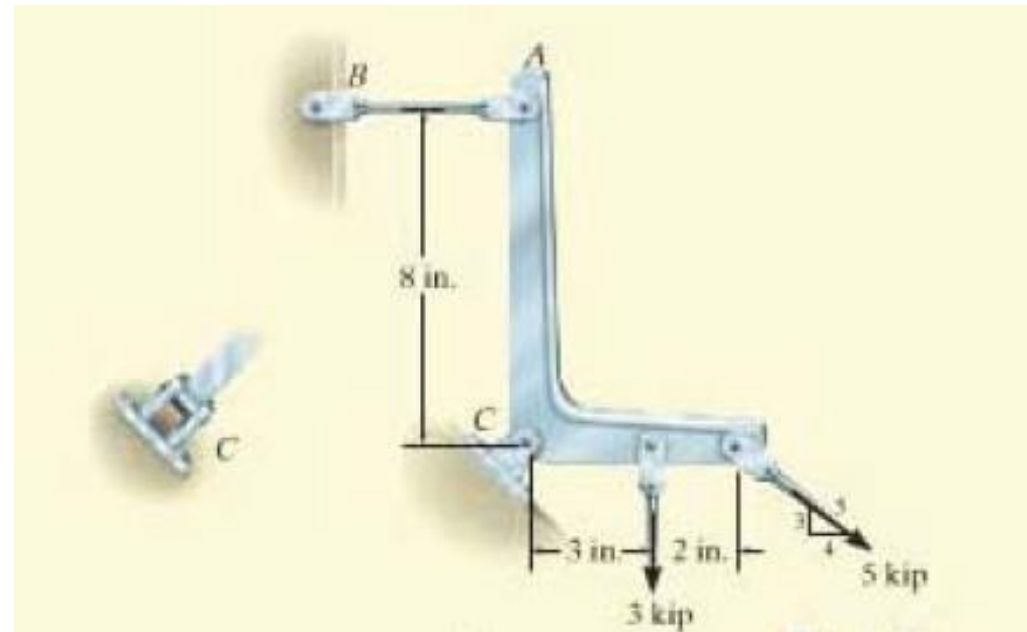
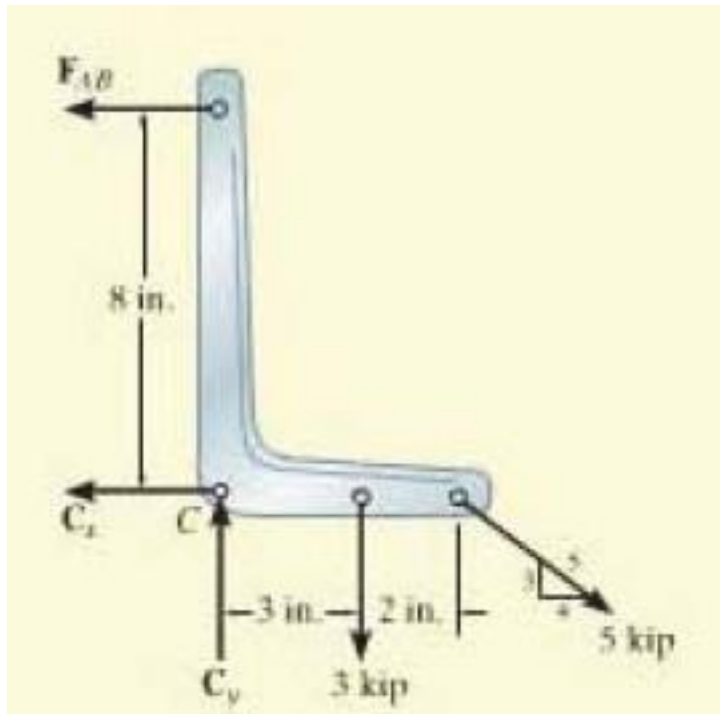
$$f .s. = \frac{\sigma_{fail}}{\sigma_{allow}}$$

$$f .s. = \frac{\tau_{fail}}{\tau_{allow}}$$

Stress

Example 1

The control arm is subjected to the loading shown in the figure. Determine to the nearest $\frac{1}{4}$ in. the required diameter of the steel pin at C if the allowable shear stress for the steel is $\tau_{\text{allow}} = 8$ ksi.



Stress

$$\sum M_C = 0 \quad \Rightarrow \quad F_{AB} * 8 - 3 * 3 - 5 \left(\frac{3}{5} \right) * 5 = 0$$

$$F_{AB} = 3 \text{ kip}$$

$$\sum F_x = 0 \quad \Rightarrow \quad -3 - C_x + 5 \left(\frac{4}{5} \right) = 0$$

$$C_x = 1 \text{ kip}$$

$$\sum F_y = 0 \quad \Rightarrow \quad C_y - 3 - 5 \left(\frac{3}{5} \right) = 0$$

$$C_y = 6 \text{ kip}$$

$$F_C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1)^2 + (6)^2} = 6.082 \text{ kip}$$

Stress

$$\tau_c = \frac{F_c}{2A} \leq \tau_{allow} \quad \Rightarrow \quad \frac{6.082}{2A} \geq 8 \quad \Rightarrow \quad A \geq 0.3801 \text{ in}^2$$

$$A = \frac{\pi}{4} d^2 \quad \Rightarrow \quad d \geq \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 * 0.3801}{\pi}} = 0.6956 \text{ in}$$

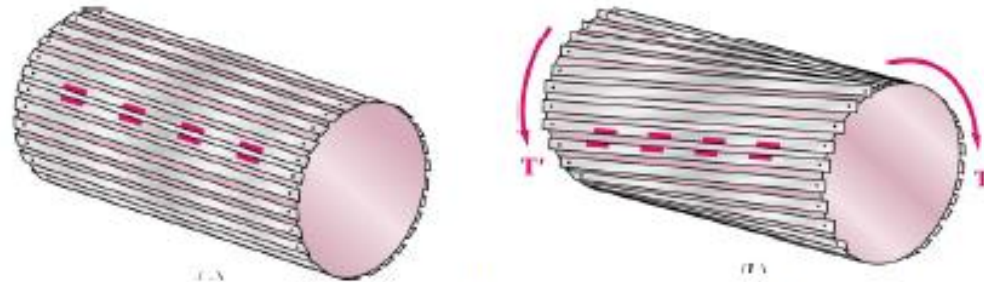
$$d \geq 0.6956 \text{ in}$$

Use a pin having a diameter of

$$d = \frac{3}{4} = 0.75 \text{ in}$$

Torsion Stress

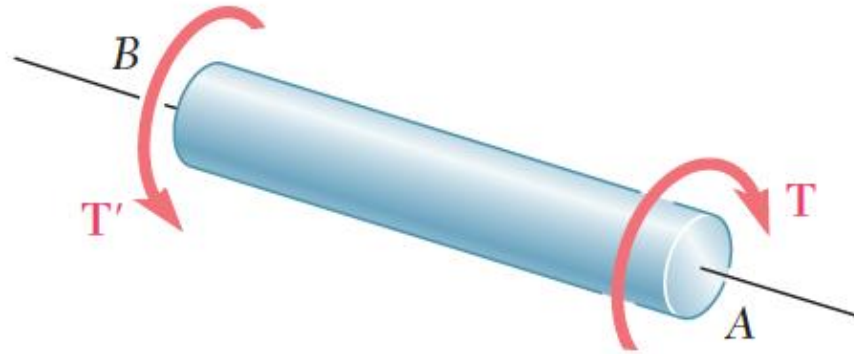
- We assume
 - Bar is in pure torsion
 - Small rotations (the length and radius will not change)
- How does the bar deform?
 - Cross-section of the bar remains the same shape, bar is simply rotating.



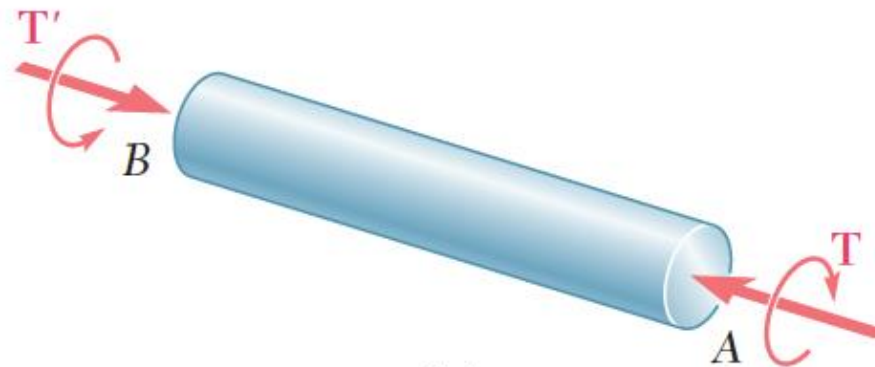
- Cross-section remains perpendicular to axis of cylinder (cylinder does not warp).



Torsion Stress



(a)



(b)

Fig. 3.1 Two equivalent ways to represent a torque in a free-body diagram.

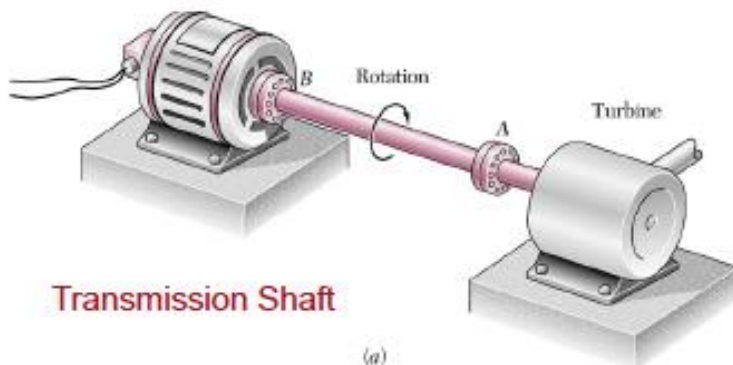
Torsion Stress

Torsion of Circular Shafts

- In this chapter, we will examine uniaxial bars subject to torque.

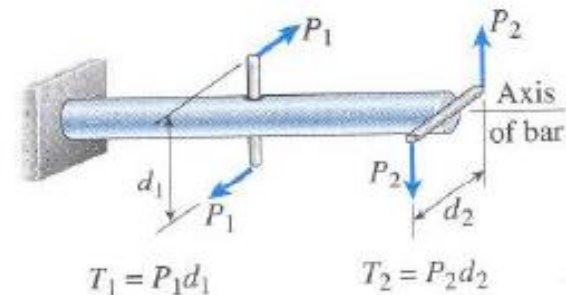


- Where does this occur?



Transmission Shaft

(a)



$$T_1 = P_1 d_1$$

$$T_2 = P_2 d_2$$

Force Couples

Torsion Stress

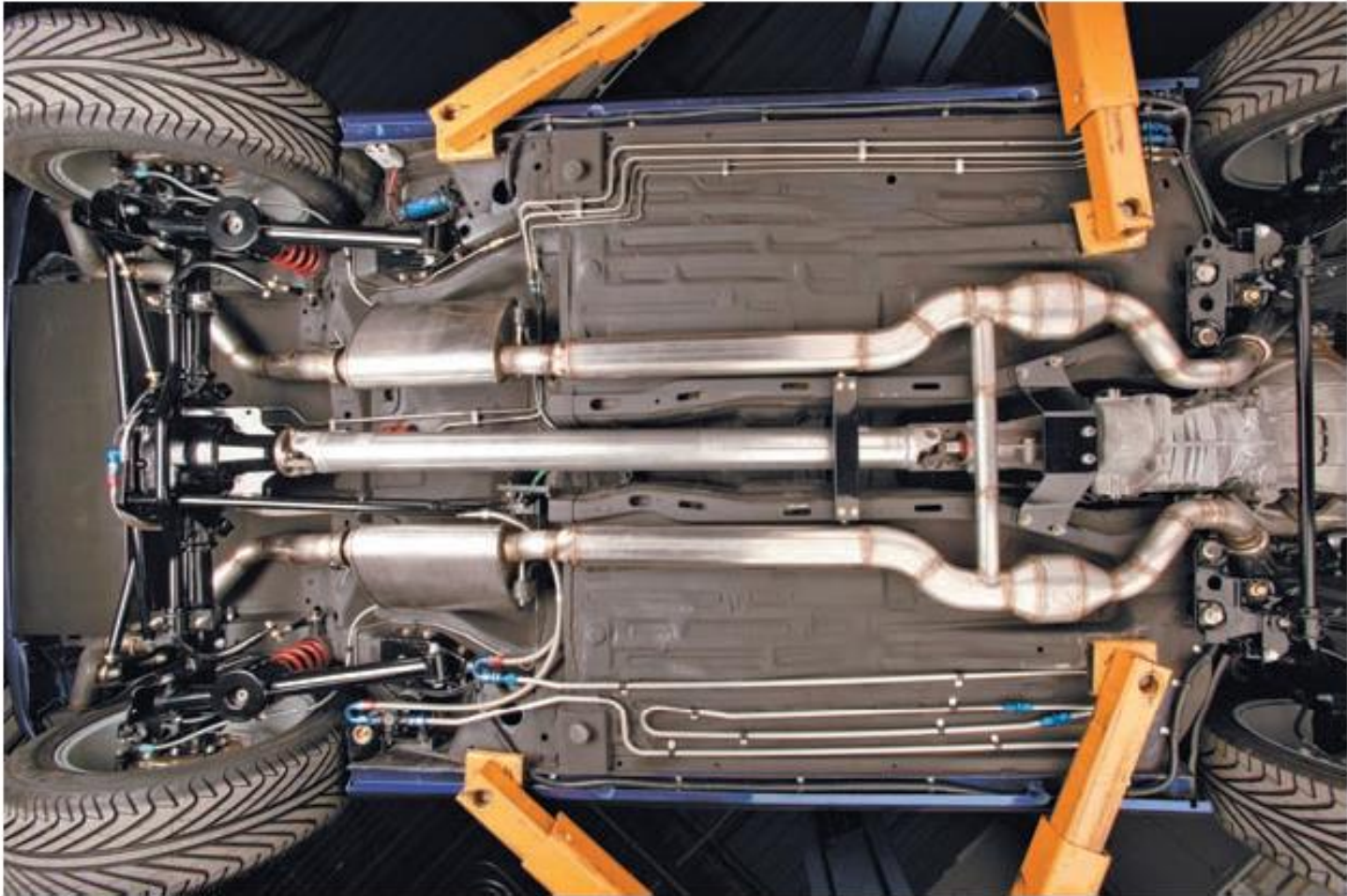
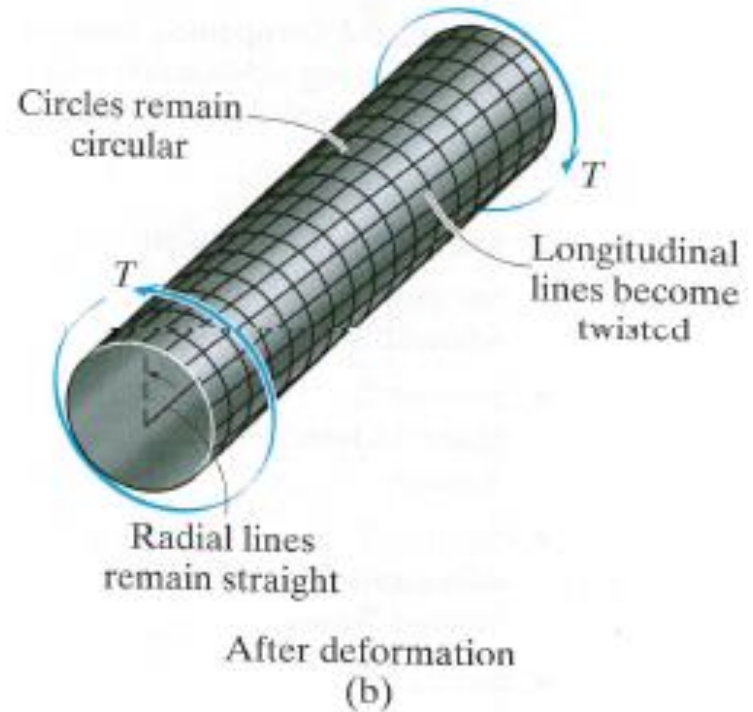
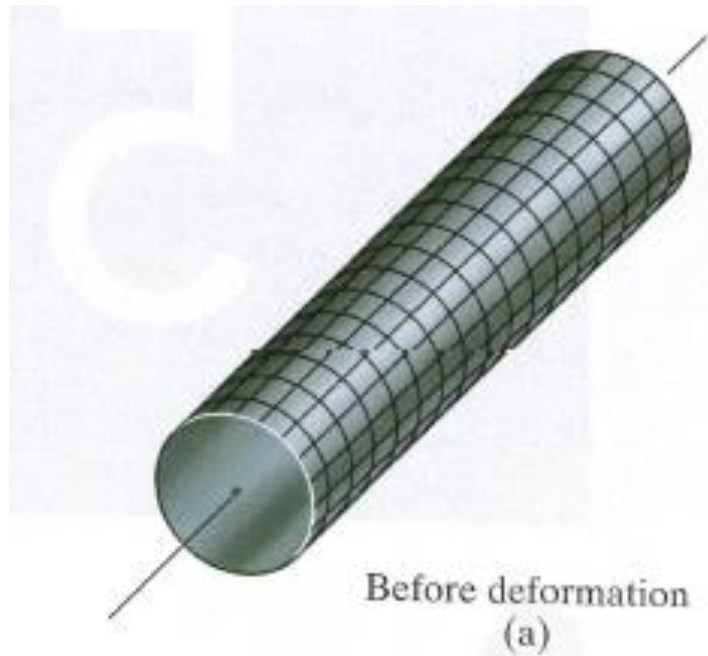
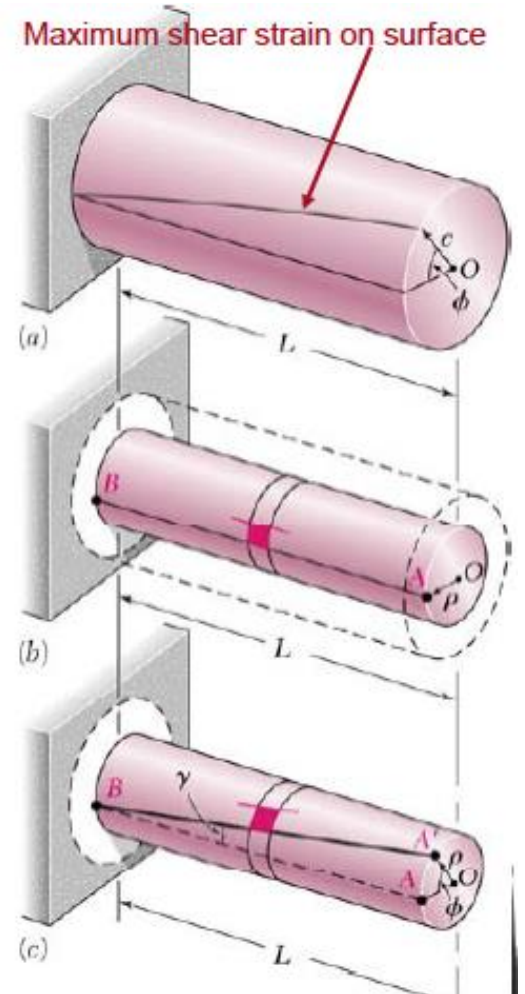
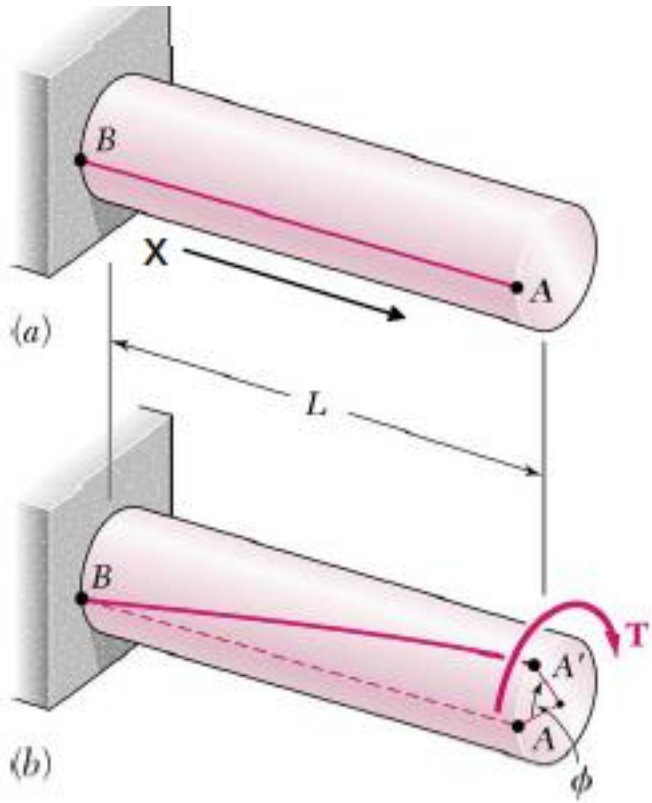


Photo 3.1 In this automotive power train, the shaft transmits power from the engine to the rear wheels.

Torsional Deformation of a Circular Shaft



Angle of Twist



The Torsion Formula

If the material is linear-elastic, then Hook's law applies, $\tau = G\gamma$, and consequently a linear variation in shear strain, leads to a linear variation in shear stress along any radial line on the cross section.

$$\tau = \frac{T\rho}{J} \quad \text{and} \quad \tau_{\max} = \frac{Tc}{J}$$

Where

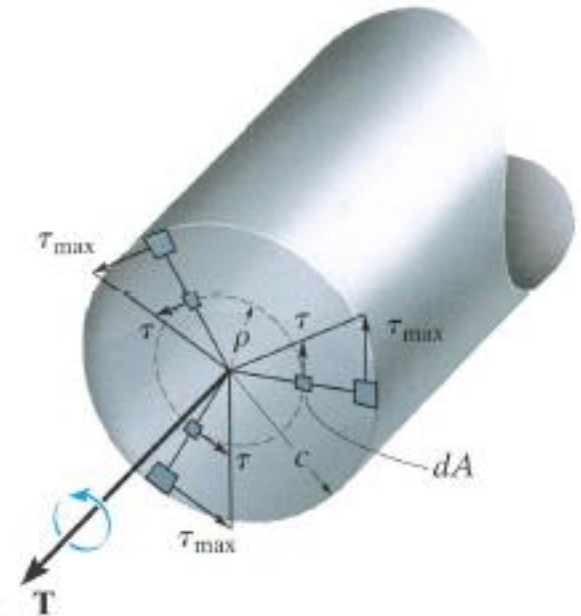
T the resultant internal torque acting at the cross section.

J the polar moment of inertia of the cross-sectional area.

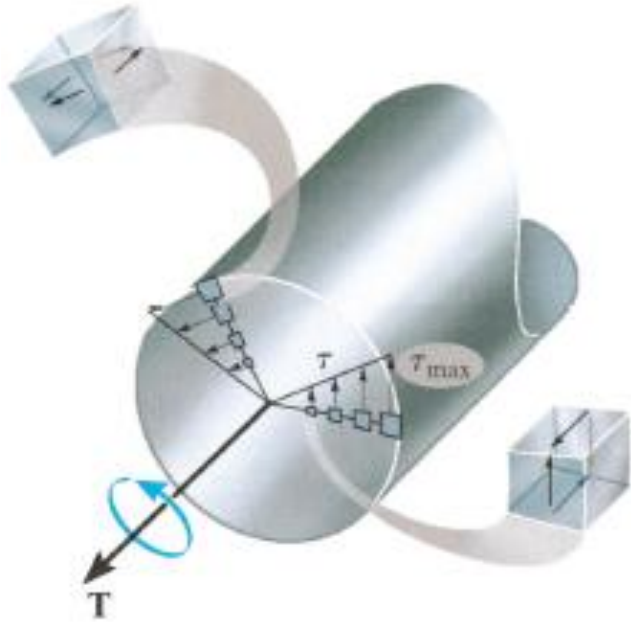
c the outer radius of the shaft.

τ_{\max} the maximum shear stress in the shaft, which occurs at the outer surface.

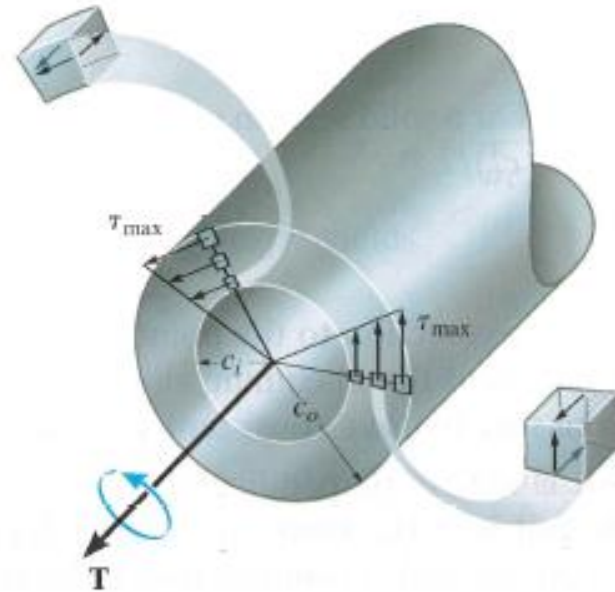
ρ the radius



The Torsion Formula



$$J = \frac{\pi}{2} c^4$$
$$= \frac{\pi}{32} d^4$$



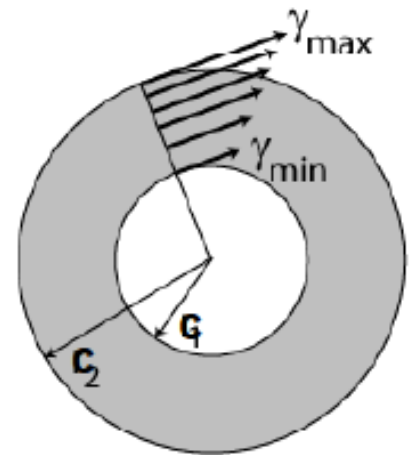
$$J = \frac{\pi}{2} (c_o^4 - c_i^4)$$
$$= \frac{\pi}{32} (d_o^4 - d_i^4)$$

Shearing Strain

- We can also apply the equation for maximum surface shear strain to a hollow circular tube.

$$\gamma_{\min} = \frac{c_1 \phi}{L}$$

$$\gamma_{\max} = \frac{c_2 \phi}{L}$$



- This applies for all types of materials: elastic, linear, non-linear, plastic, etc.

Torque stress summary

Shear Stress

$$\tau = G\gamma = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$\phi = \frac{TL}{GJ}$$

τ : Shear Stress

r : shaft radius

G : Modulus of rigidity

d : Shaft diameter

γ : Shear Strain

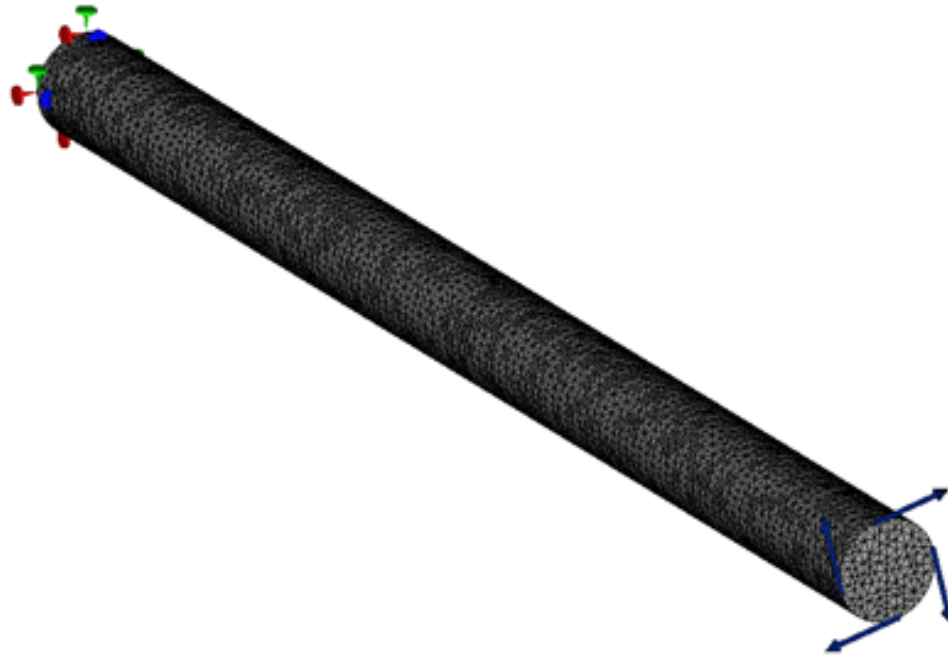
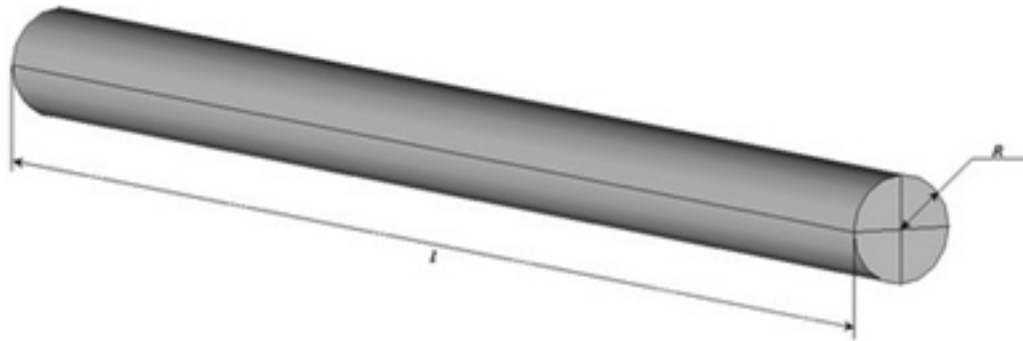
J : Polar moment of inertia

For a hollow circular shaft of inner radius c_1 and outer radius c_2 , the polar moment of inertia is

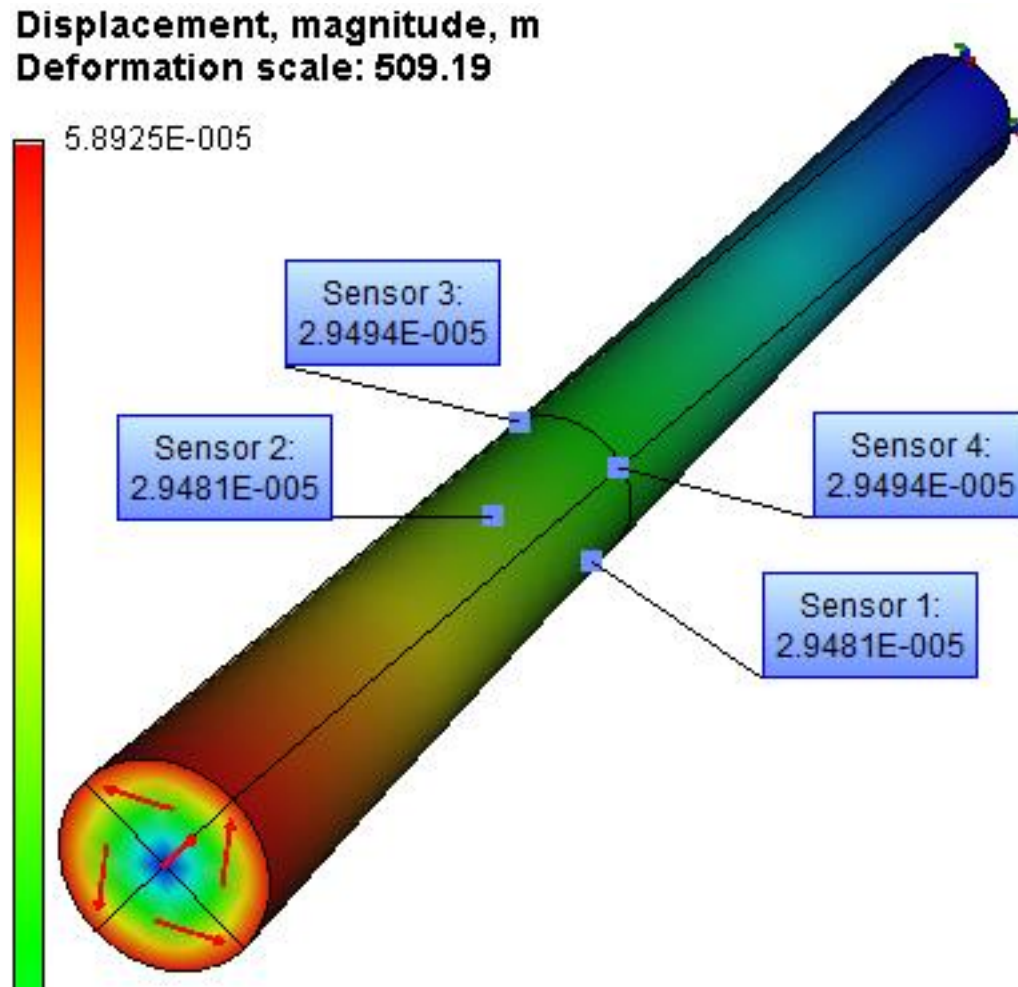
$$J = \frac{\pi}{32} (d_o^4 - d_i^4)$$

ϕ : Angle of Twist

Torsion of a Shaft with Circular Cross-Section Finite Element Analysis



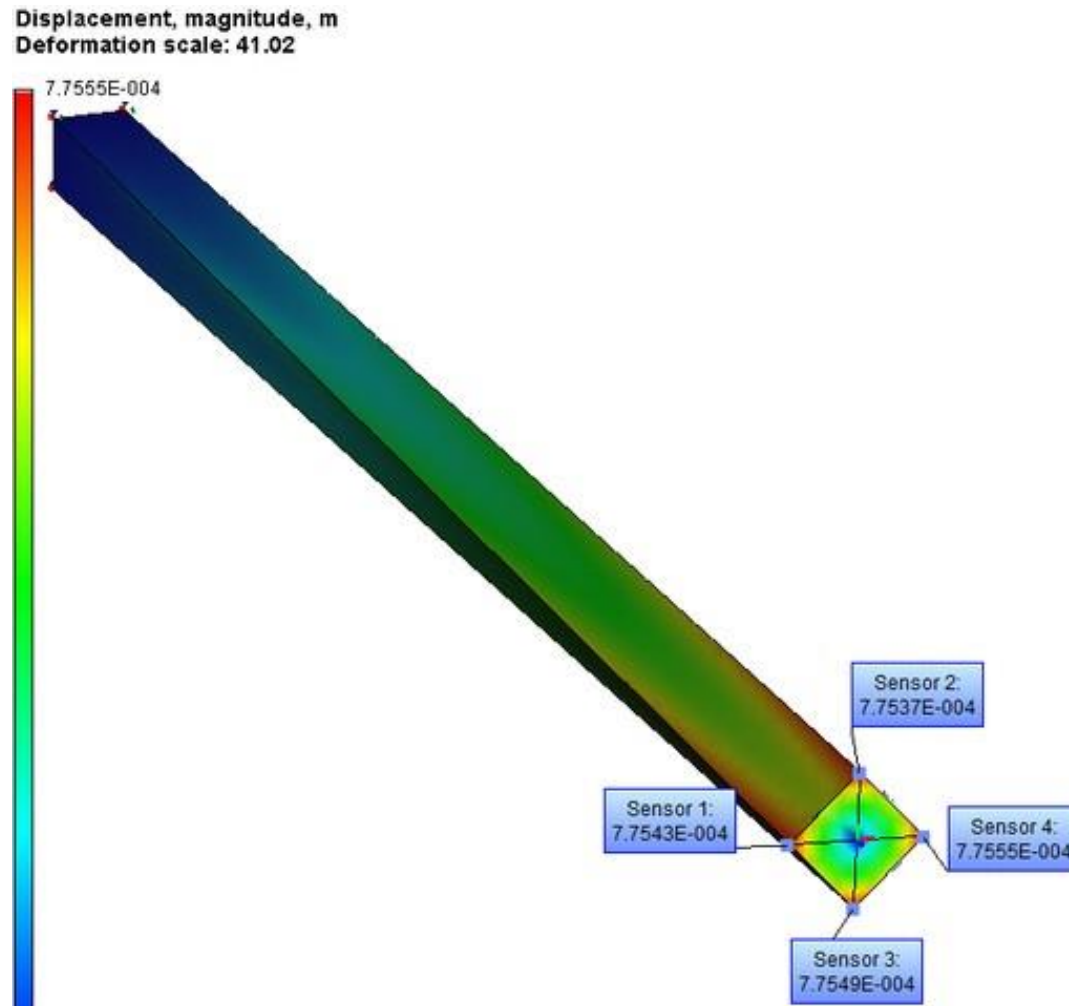
Torsion of a Shaft with Circular Cross-Section Finite Element Analysis



Torsion of a Beam with the Square Cross-Section Finite Element Analysis



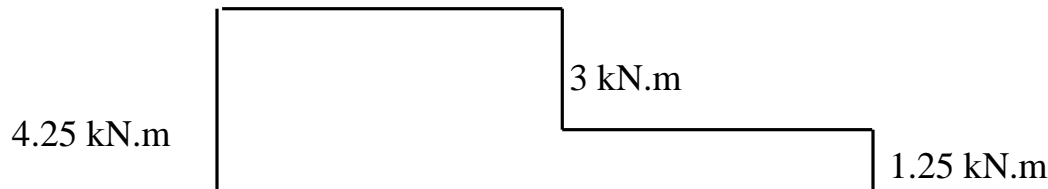
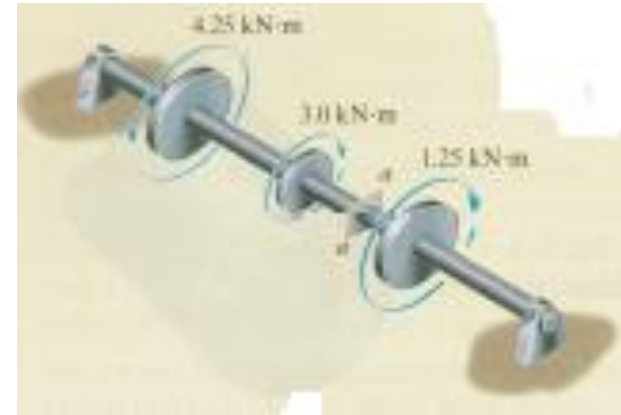
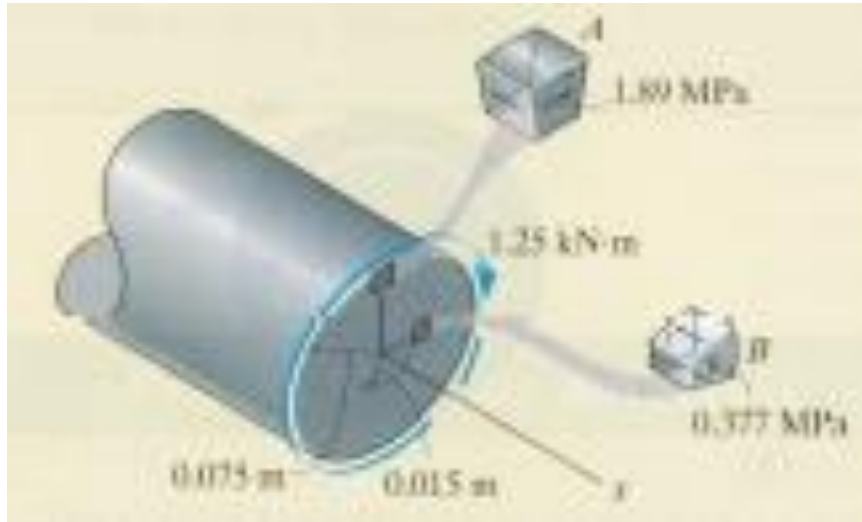
Torsion of a Beam with the Square Cross-Section Finite Element Analysis



The Torsion Formula

Example 1

The shaft shown in the attached figure is supported by two bearings and is subjected to three torques. Determine the shear stress developed at points *A* and *B* located at section *a-a* of the shaft. The shaft diameter is 75 mm.



Torque diagram

The Torsion Formula

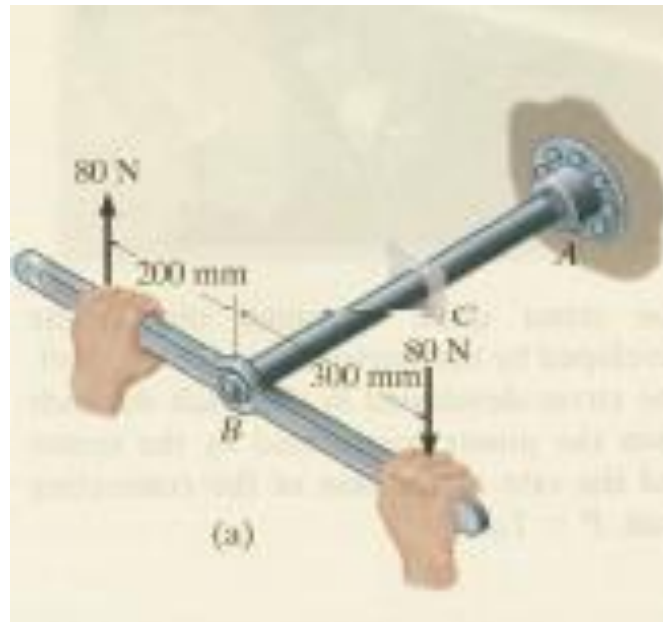
$$\tau_A = \frac{Tr}{J} = \frac{1.25 * 1000 * 1000 * 750 / 2}{\frac{\pi}{32} (750)^4} = 1.89 \text{ MPa}$$

$$\tau_B = \frac{Tr}{J} = \frac{1.25 * 1000 * 1000 * 150 / 2}{\frac{\pi}{32} (750)^4} = 0.337 \text{ MPa}$$

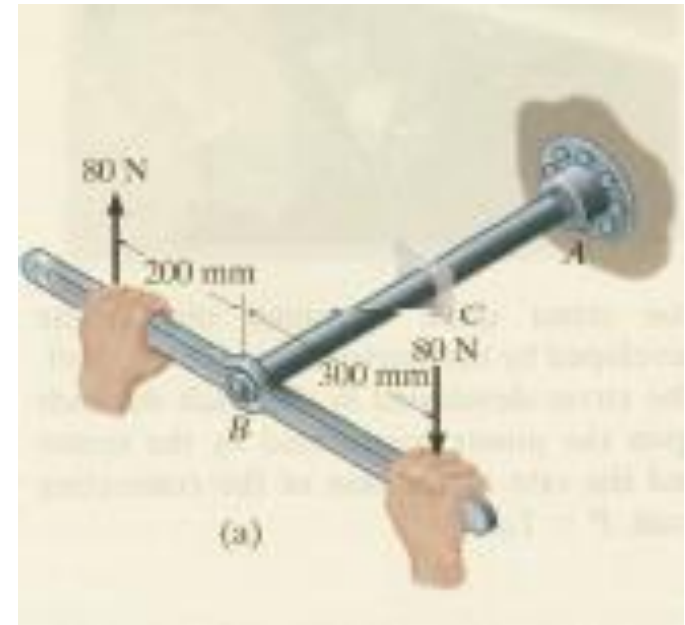
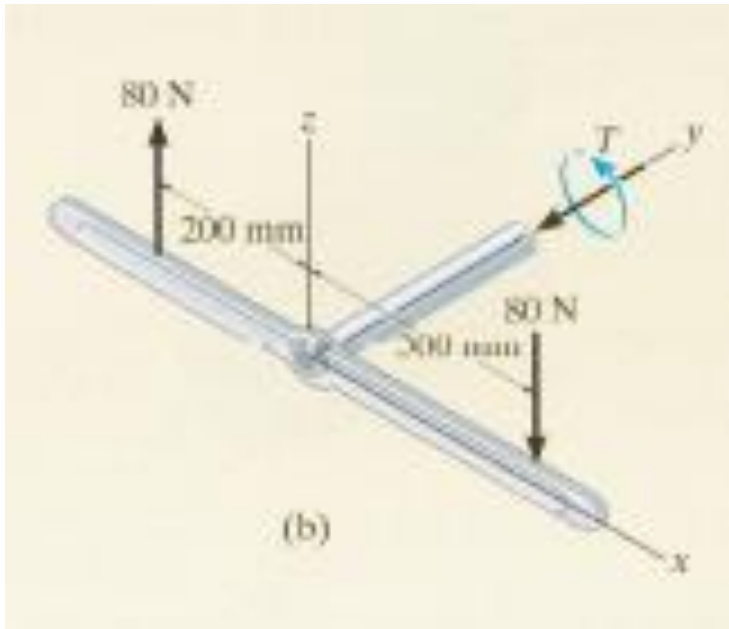
The Torsion Formula

Example 2

The pipe shown in the attached figure has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at A using a torque wrench at B , determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.



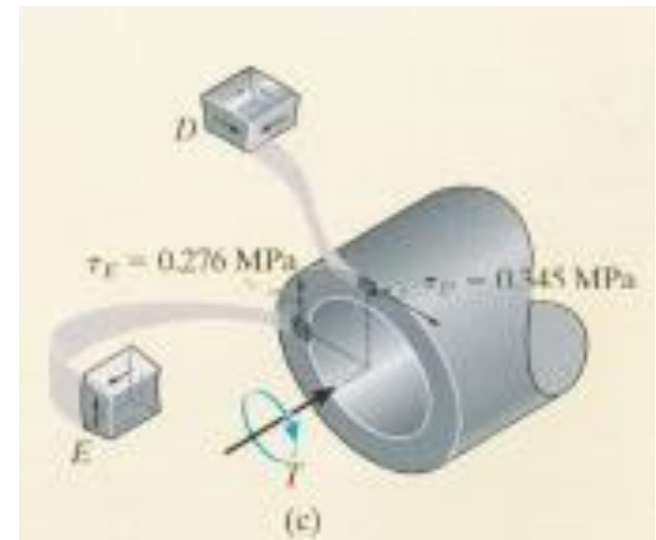
The Torsion Formula



$$\sum M_y = 0; \quad 80 \cdot 0.3 + 80 \cdot 0.2 - T = 0$$

$$T = 40 \text{ N.m}$$

$$\tau_o = \frac{Tc_o}{J} = \frac{40000 \cdot 50}{\frac{\pi}{2}(50^4 - 40^4)} = 0.345 \text{ MPa}$$



The Power Transmission

$$\tau_i = \frac{Tc_i}{J} = \frac{40000 * 40}{\frac{\pi}{2} (50^4 - 40^4)} = 0.276 \text{ MPa}$$

The Power Transmission

Shafts and tubes having circular cross sections are often used to transmit power developed by a machine. When used for this purpose, they are subjected to a torque that depends on the power generated by the machine and the angular speed of the shaft.

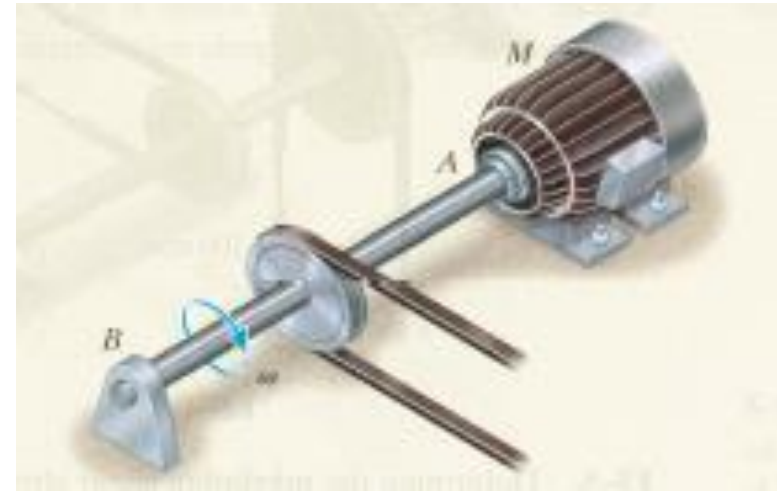
$$P = T\omega$$



The Power Transmission

Example 1

A solid steel shaft AB shown in the figure is to be used to transmit 3750 W from the motor M to which it is attached. If the shaft rotates at $\omega = 175$ rpm and the steel has an allowable shear stress of $\tau_{\text{allow}} = 100$ MPa determine the required diameter of the shaft to the nearest mm.



$$P = T\omega$$

$$T = \frac{3750 * 1000}{\frac{2\pi * 175}{60}} = 204627.8 \text{ N.mm}$$

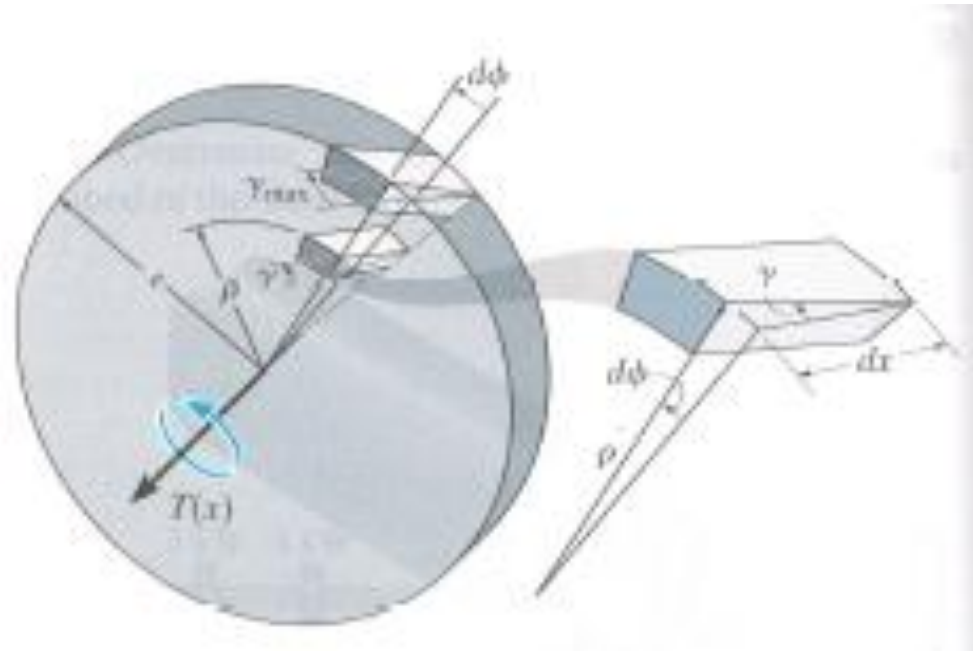
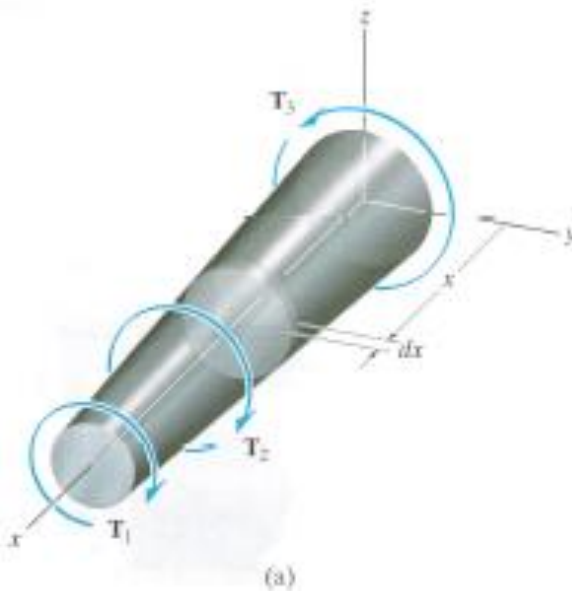
$$\tau_{\text{max}} = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3} \leq \tau_{\text{allow}}$$

The Angle of Twist

$$\frac{16(204627.8)}{\pi d^3} \leq 100 \quad \Rightarrow \quad d \geq 21.843 \text{ mm} \quad \Rightarrow \quad d = 22 \text{ mm}$$

Angle of Twist

Occasionally the design of a shaft depends on restricting the amount of rotation or twist that may occur when the shaft is subjected to a torque. Furthermore, being able to compute the angle of twist for a shaft is important when analyzing the reactions on statically indeterminate shafts.



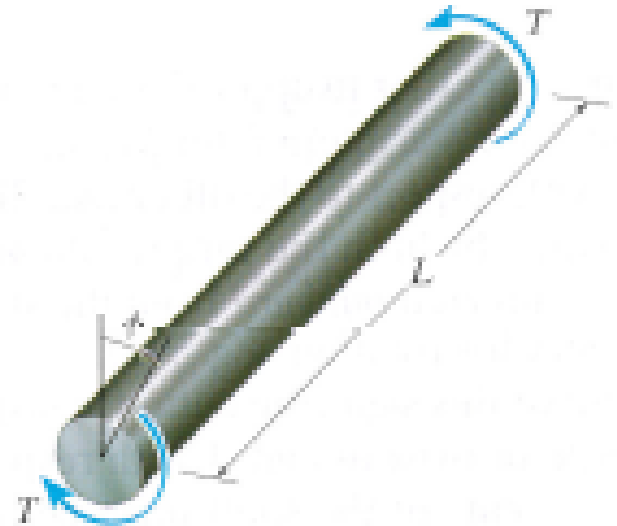
The Angle of Twist

$$d\phi = \gamma dx; \quad \gamma = \frac{\tau}{G} \quad \text{and} \quad \tau = \frac{T\rho}{J}$$
$$d\phi = \frac{T(x)}{J(x)G} dx \quad \Rightarrow \quad \phi = \int_0^L \frac{T(x)}{J(x)G} dx$$

Constant Torque and Cross-Sectional Area

$$\phi = \frac{TL}{JG}$$

The similarities between the above equations and those for an axially loaded members should be noted.

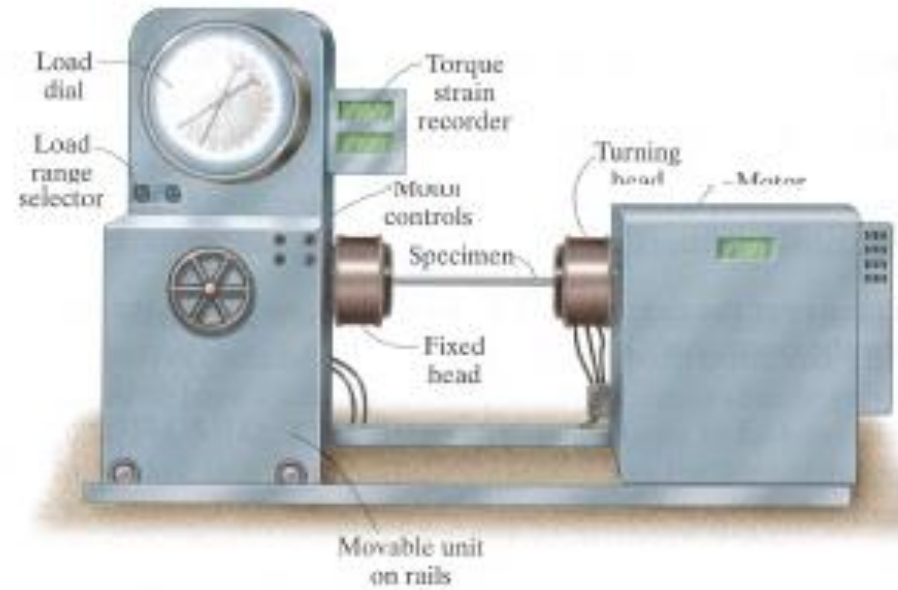


The Angle of Twist

The equation of angle of twist is often used to determine the shear modulus of elasticity G of a material. To do so, a specimen of known length and diameter is placed in a testing machine like shown in the attached figure. The applied torque T and angle of twist ϕ are then measured along the length L .

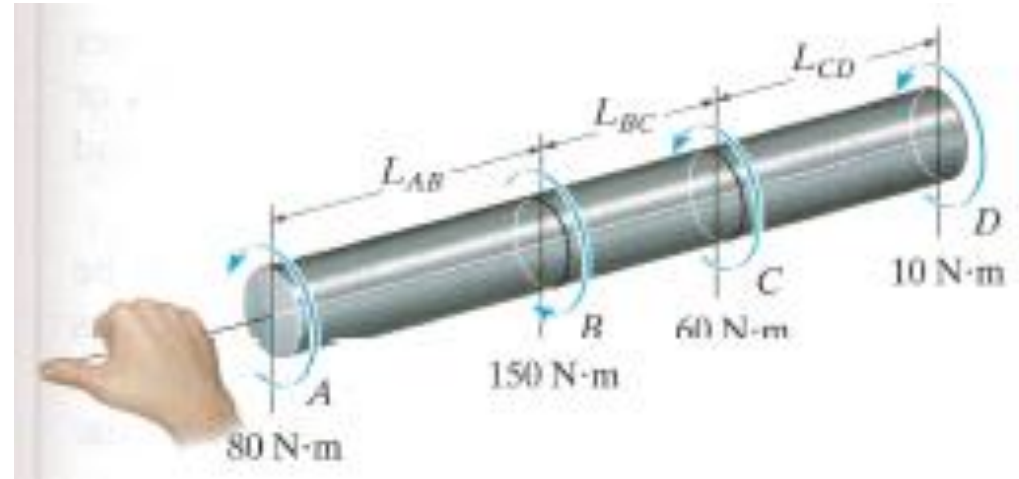
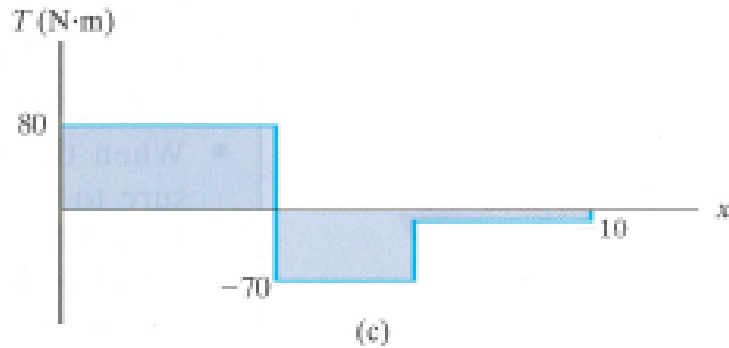
Multiple Torques

$$\phi = \sum \frac{TL}{JG}$$



The Angle of Twist

Sign Convention

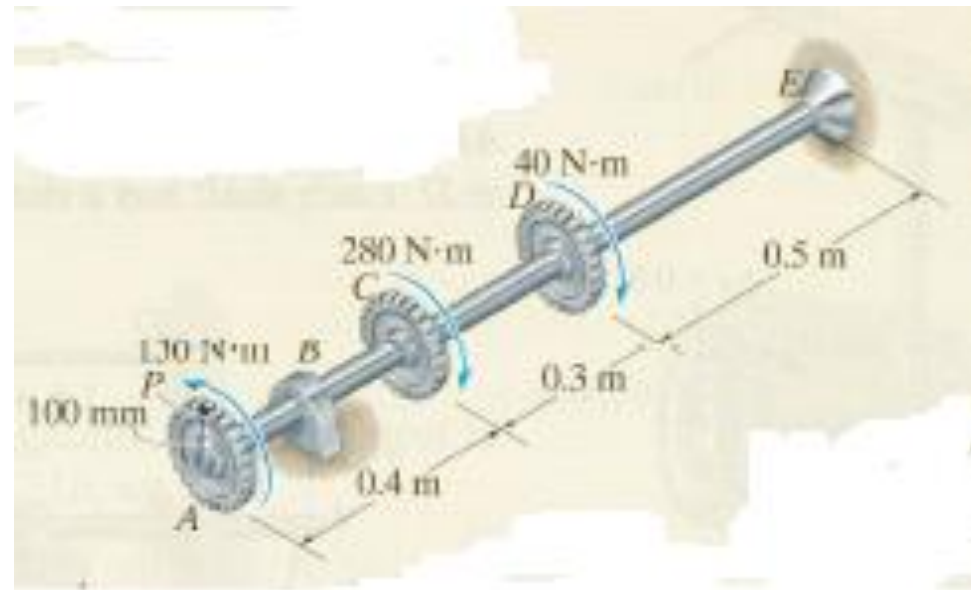
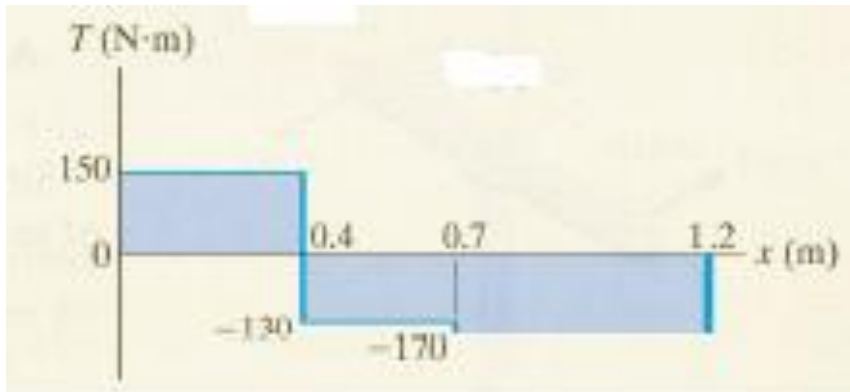


$$\phi_{A/D} = \sum \frac{TL}{JG} = \frac{(80000)L_{AB}}{JG} + \frac{(-70000)L_{BC}}{JG} + \frac{(-10000)L_{CD}}{JG}$$

The Angle of Twist

Example 1

The gears attached to the fixed-end steel shaft are subjected to the torques shown in the figure. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of tooth P on gear A . The shaft turns freely within the bearing at B .



The Angle of Twist

$$\phi_A = \sum \frac{TL}{JG} = \frac{1}{\frac{\pi}{2}(7)^4(80000)} (150000 * 400 - 130000 * 300 - 170000 * 500)$$
$$= -0.2121 \text{ rad}$$

Since the answer is negative, by the right-hand rule the thumb is directed toward the end E of the shaft and therefore gear A will rotate as shown in the attached figure.

The displacement of tooth P on gear A is

$$s_P = \phi_A r = 0.2121 * 100 = 21.2 \text{ mm}$$

