Stress Analysis

Lecture 4

ME 276

Spring 2017-2018

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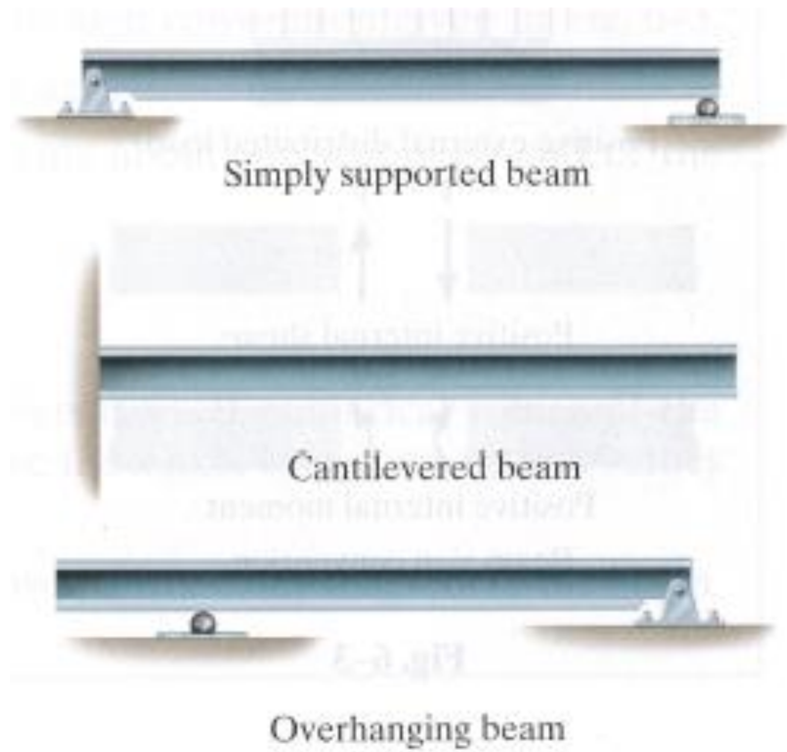
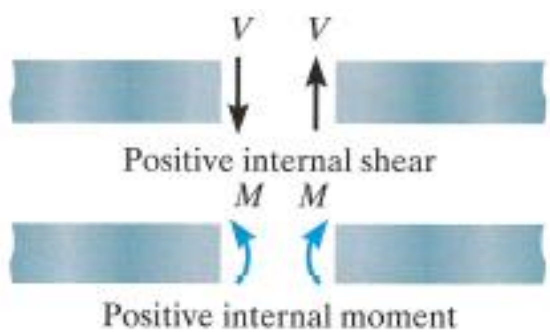
Shear and Moment Diagrams

Beam Sign Convention

The positive directions are as follows:

- The internal shear force causes a clockwise rotation of the beam segment on which it acts.
- The internal moment causes compression in the top fibers of the segment such it bends the segment so that it "hold water".



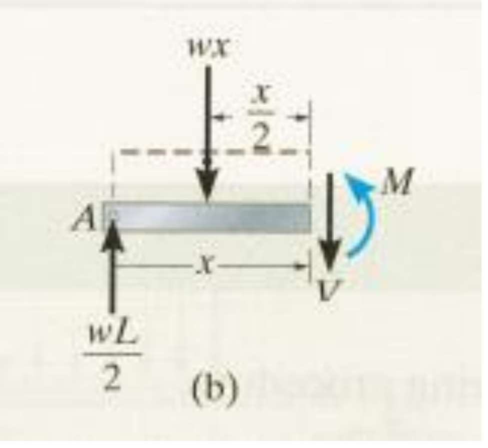


Beam sign convention

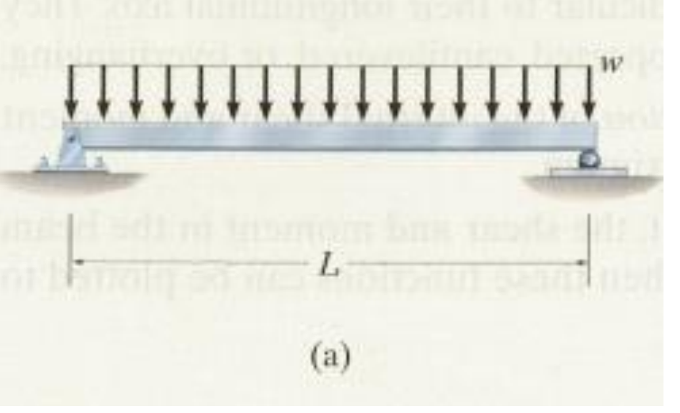
Shear and Moment Diagrams

Example 1

Draw the shear and moment diagrams for the beam shown in the attached figure.



(b)



(a)

$$\sum F_y = 0 \quad \Rightarrow \quad \frac{wL}{2} - wx - V = 0$$

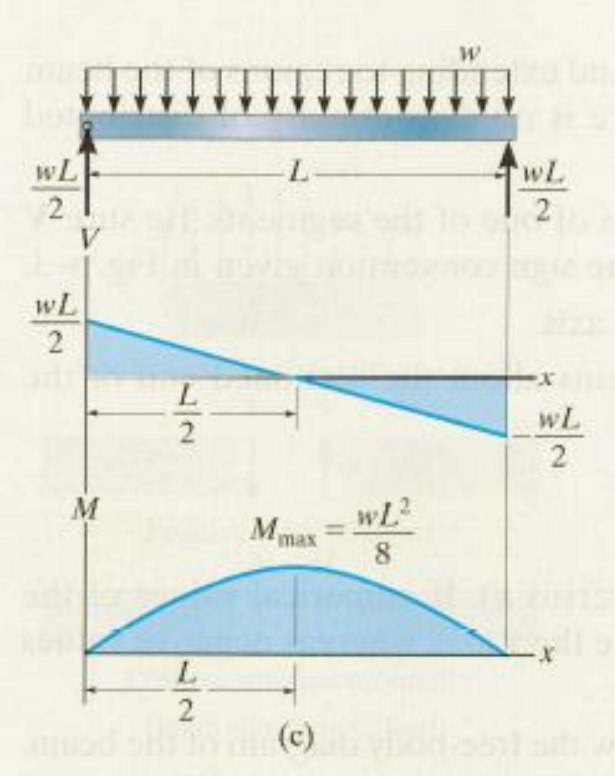
$$V = w\left(\frac{L}{2} - x\right) \qquad (1)$$

$$\sum M = 0 \quad \Rightarrow \quad \left(\frac{wL}{2}\right)x - (wx)\left(\frac{x}{2}\right) - M = 0$$

$$M = \frac{w}{2}\left(Lx - x^2\right) \qquad (2)$$

Shear and Moment Diagrams

$$M_{\max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right] = \frac{wL^2}{8}$$



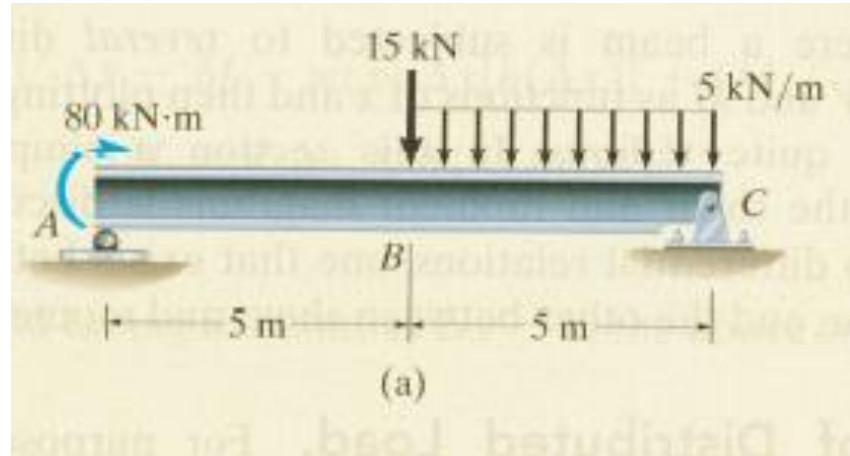
Note

$$V = \frac{dM}{dx}$$

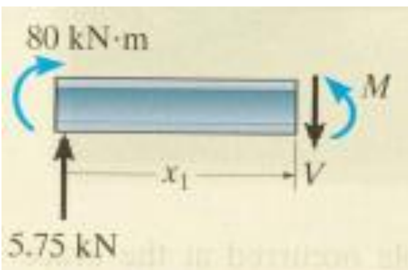
Shear and Moment Diagrams

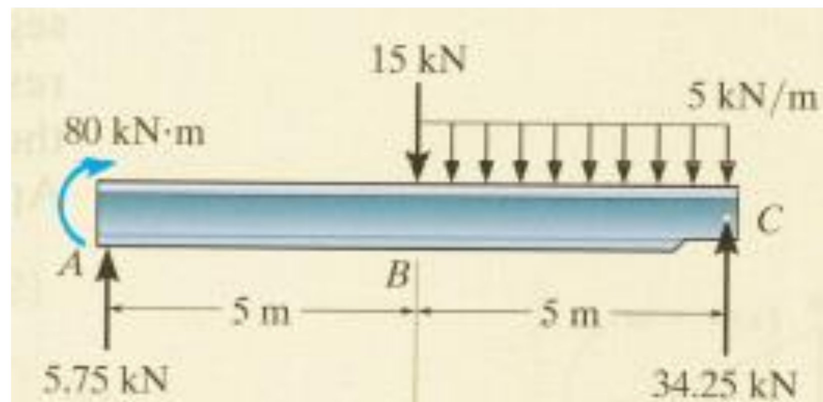
Example 2

Draw the shear and moment diagrams for the beam shown in the attached figure.



$0 \le x_1 < 5$





$$\sum F_y = 0 \quad \Rightarrow \quad 5.75 - V = 0$$

$$V = 5.75 \text{ kN} \qquad (1)$$

$$\sum M = 0 \quad \Rightarrow \quad -80 - 5.75x_1 + M = 0$$

$$M = (5.75x_1 + 80) \text{ kN.m} \qquad (2)$$

Shear and Moment Diagrams

$5 < x_2 \leq 10$

$$\sum F_y = 0$$

$$5.75 - 15 - 5(x_2 - 5) - V = 0$$

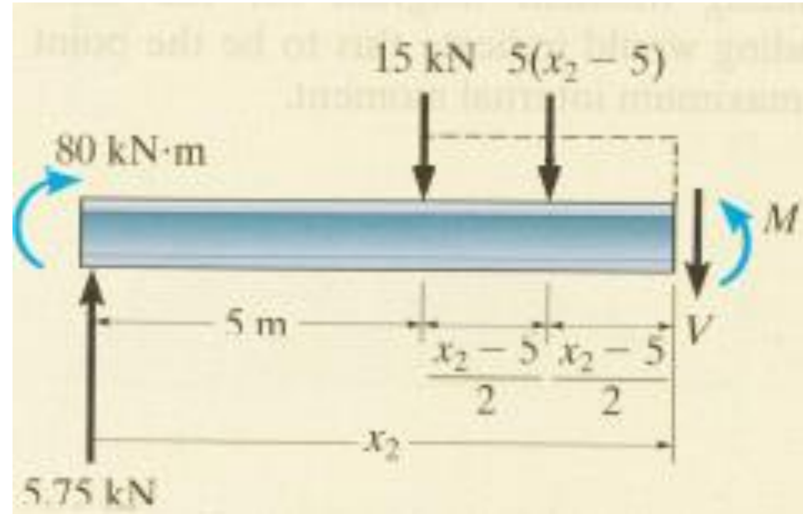
$$V = (15.75 - 5x_2) \text{ kN} \qquad (3)$$

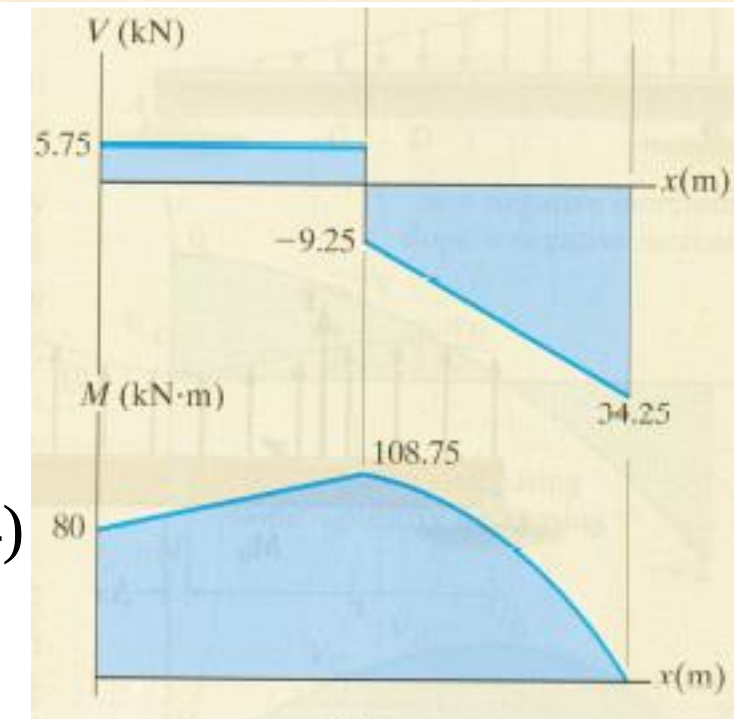
$$\sum M = 0$$

$$-80 - 5.75x_2 + 15(x_2 - 5)$$

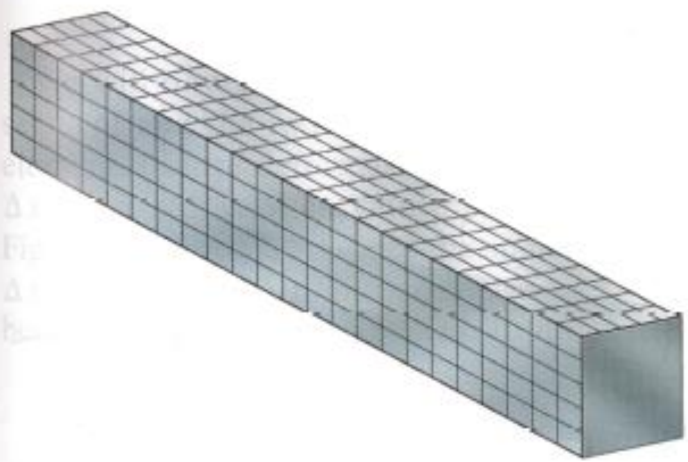
$$+ 5(x_2 - 5)\left(\frac{x_2 - 5}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5) \text{ kN.m} \qquad (4)$$



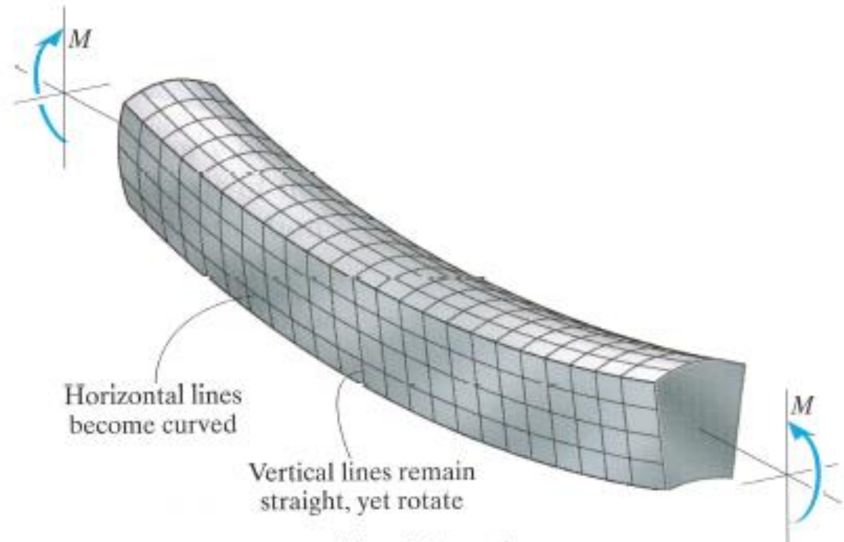


The Bending Formula



Before deformation

(a)

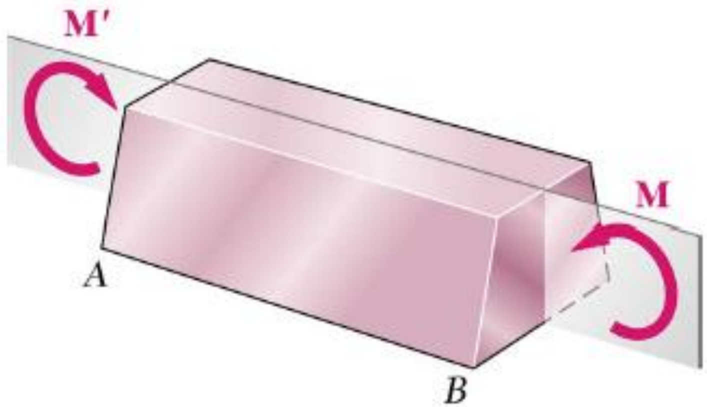


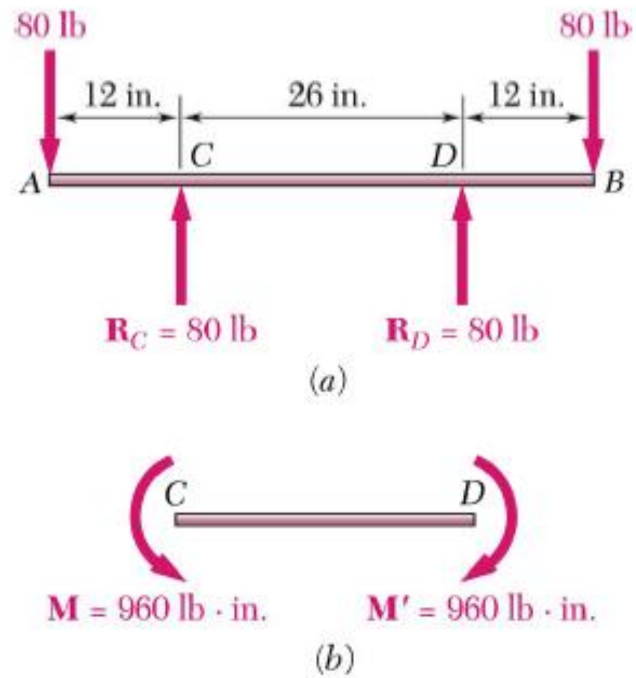
After deformation

(b)

The Bending Formula

- Prismatic beams subject to equal and opposite couples acting in the same plane are in pure bending.



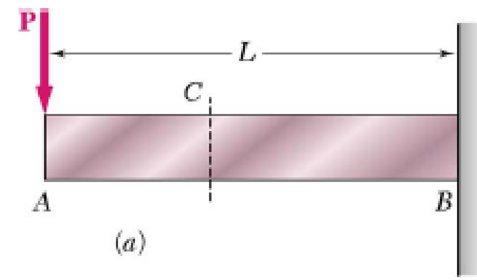


The Bending Formula

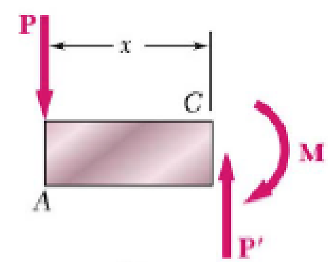
Pure vs. Non-Uniform Bending

- Pure bending: Shear force (V) = 0 over the section
- Non-uniform bending: V ≠ 0 over the section

Non-uniform bending
- Moment → normal stresses
- Shear force → shear stresses



(a)



(b)

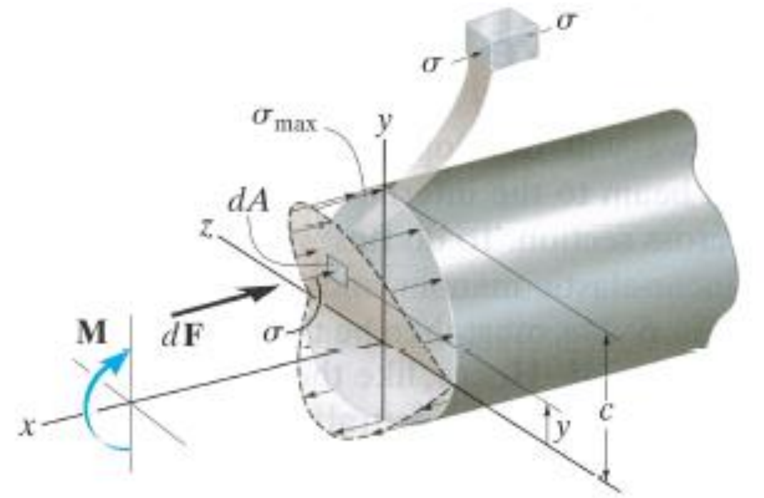
Pure bending
- Moment → normal stresses

(b)

The Bending Formula

$$\sigma = \frac{M}{I} y$$

$$\sigma_{\max} = \frac{M}{I} c$$



Bending stress variation

Where

M the resultant internal moment acting at the cross section.

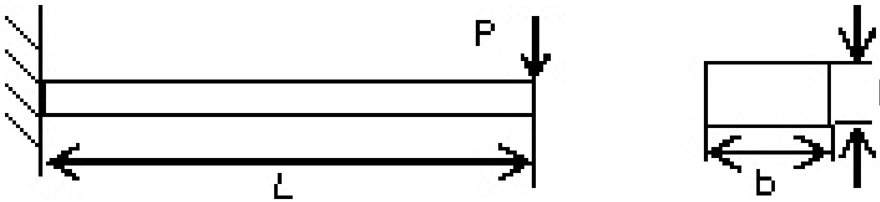
I the moment of inertia of the cross-sectional area about the neutral axis.

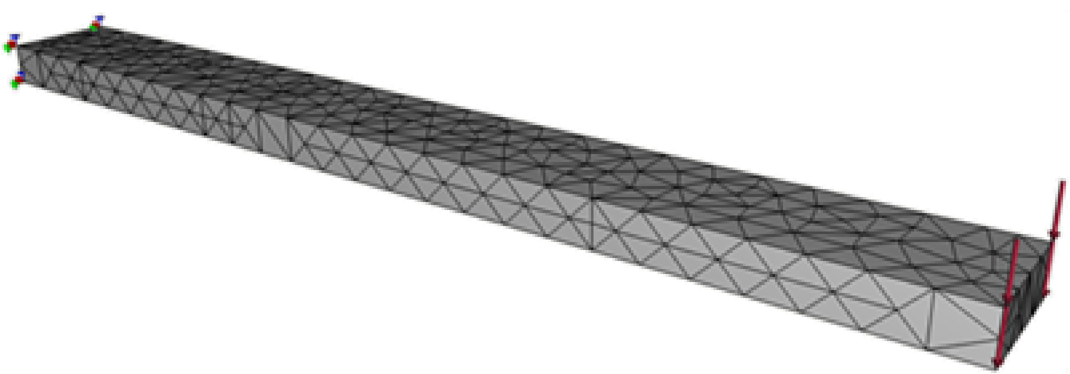
c perpendicular distance from the neutral axis to a point farthest away from the neutral axis. This is where σ_{max} acts.

σ_{max} the maximum bending stress in the shaft, which occurs at the outer surface.

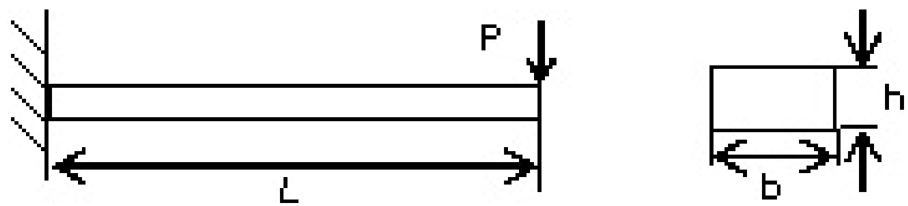
y perpendicular distance from the neutral axis to a point where σ is to be calculated

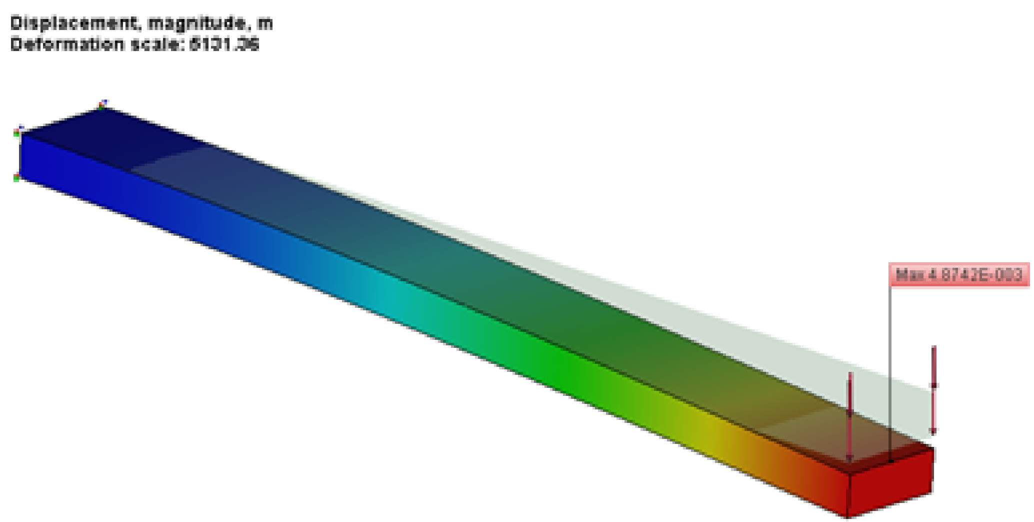
Bending a Cantilever Beam under a Concentrated Load



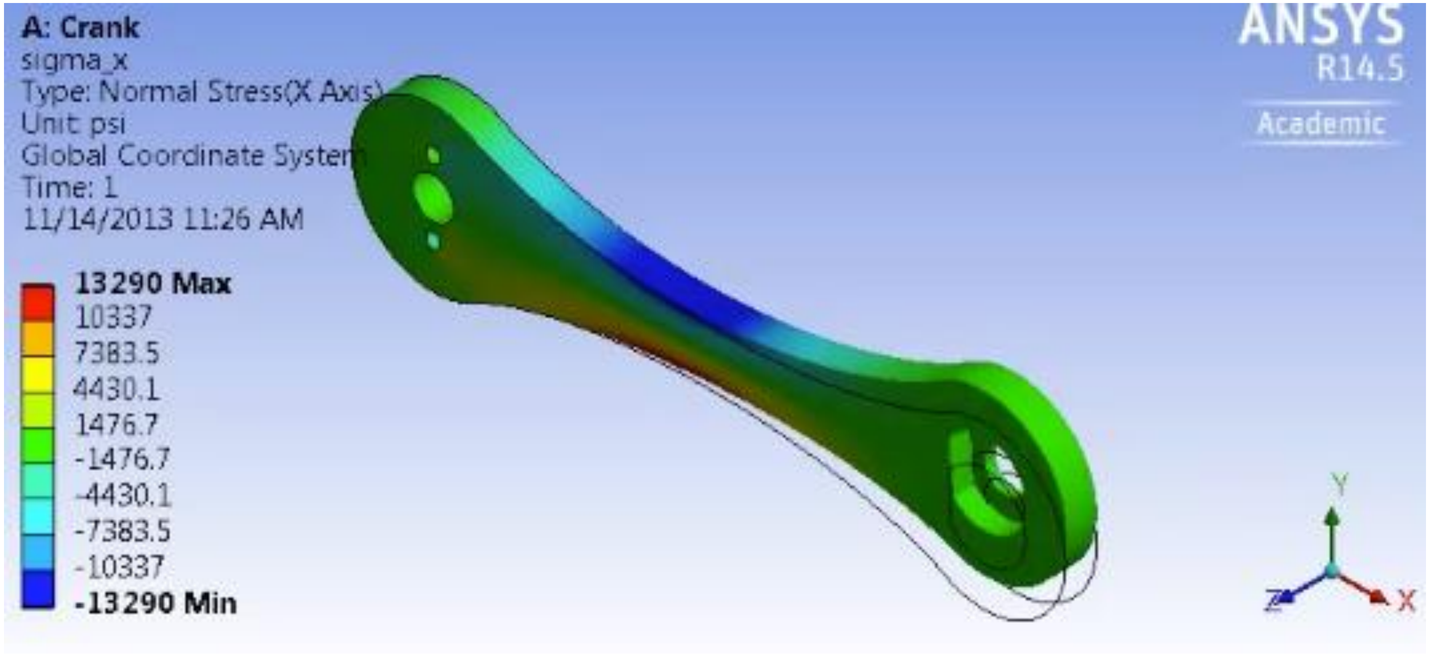


Bending a Cantilever Beam under a Concentrated Load

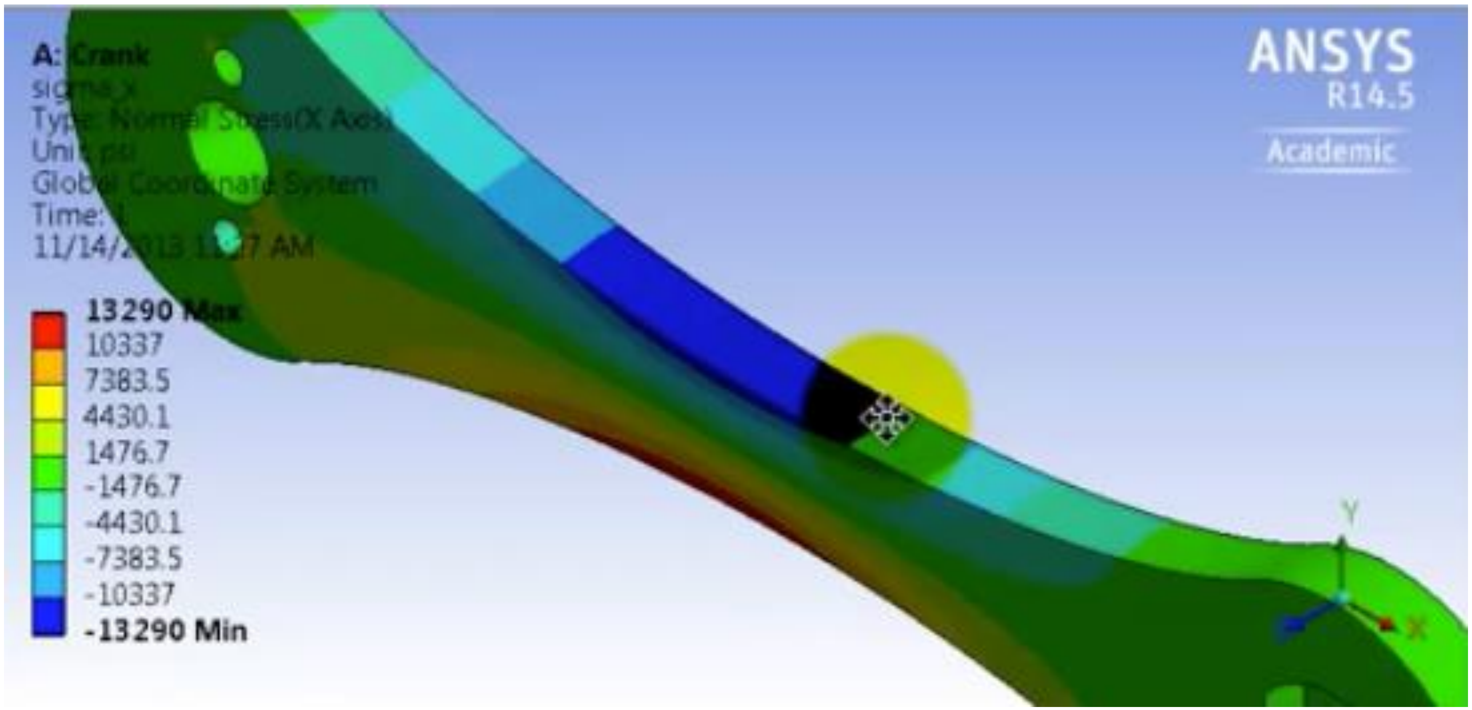




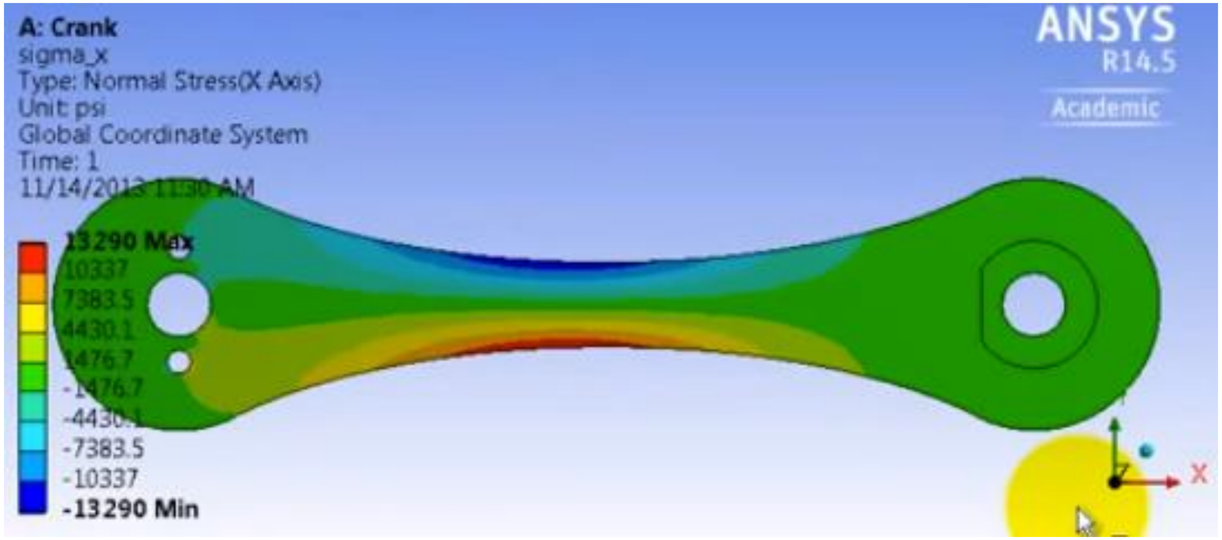
Bending Stress

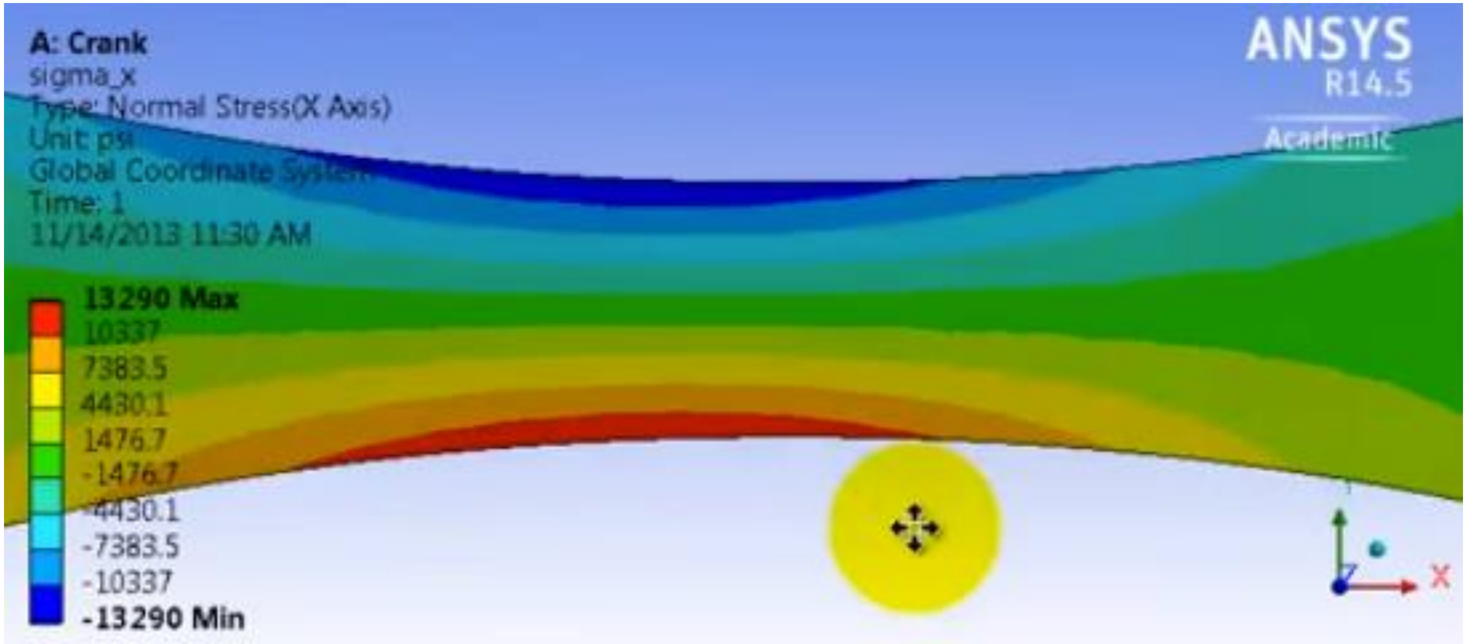


Bending Stress

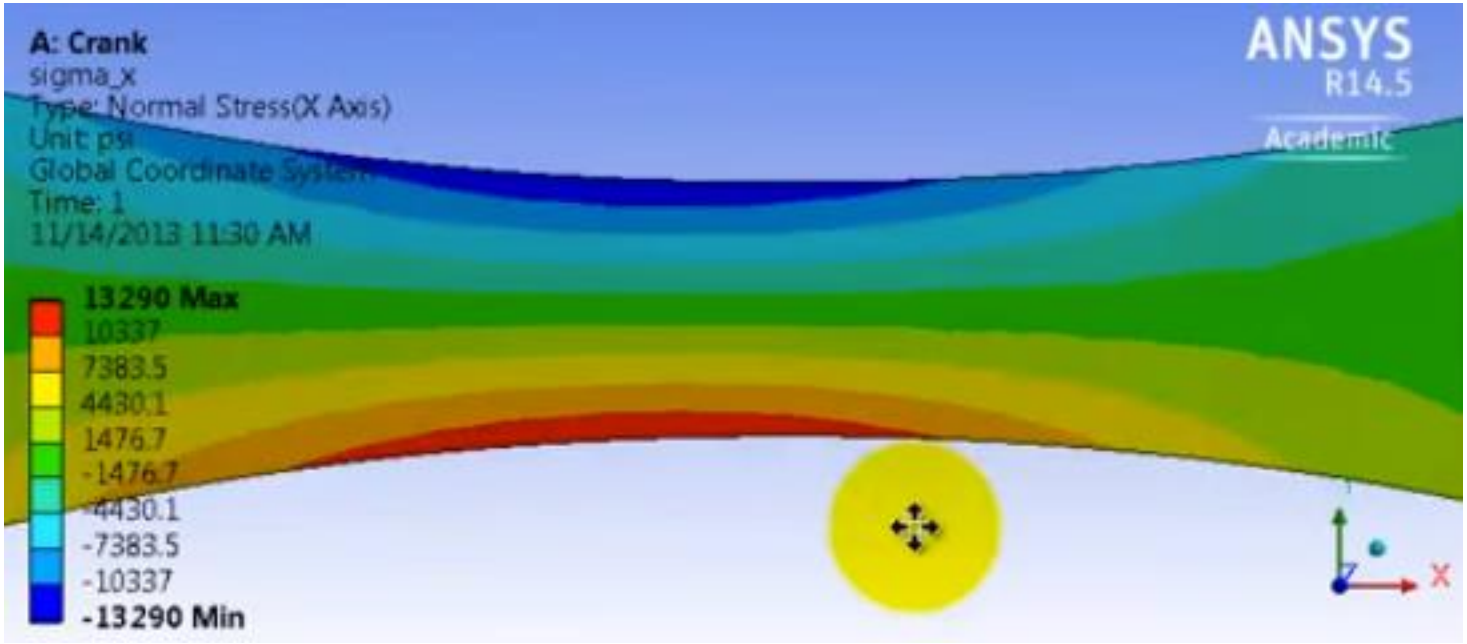


Bending Stress

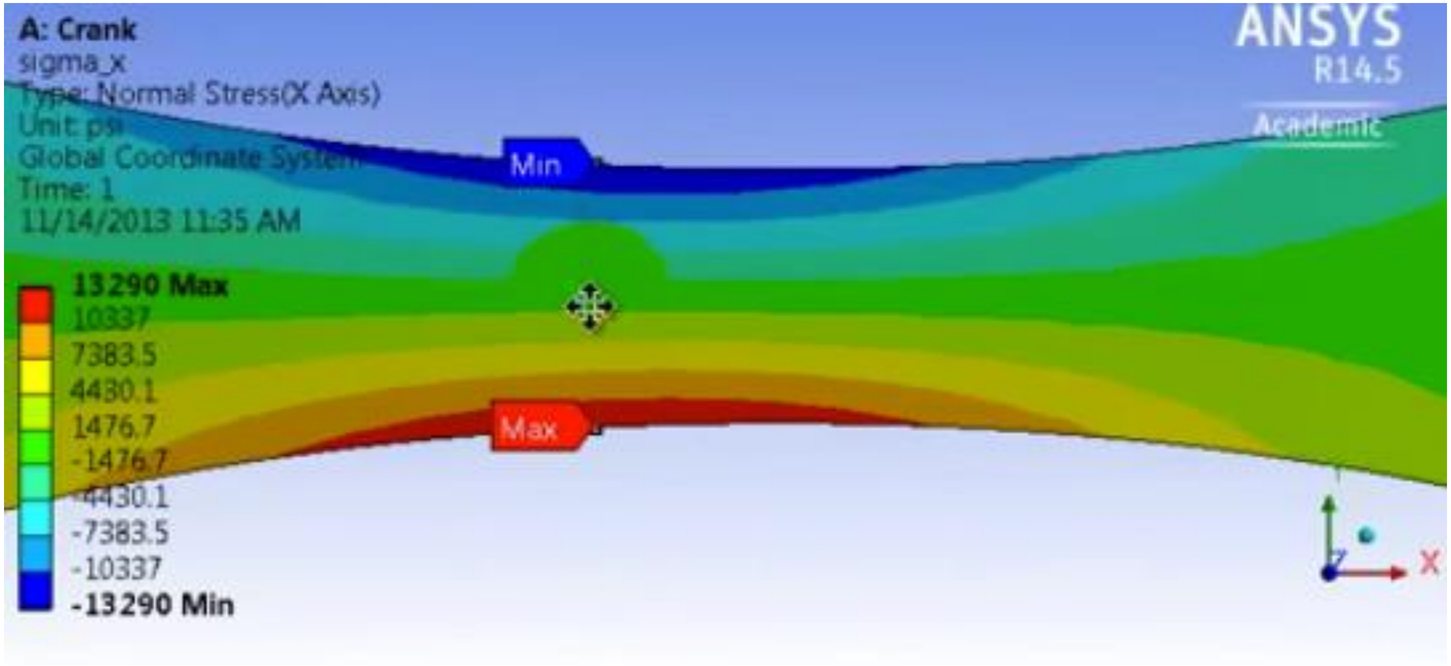




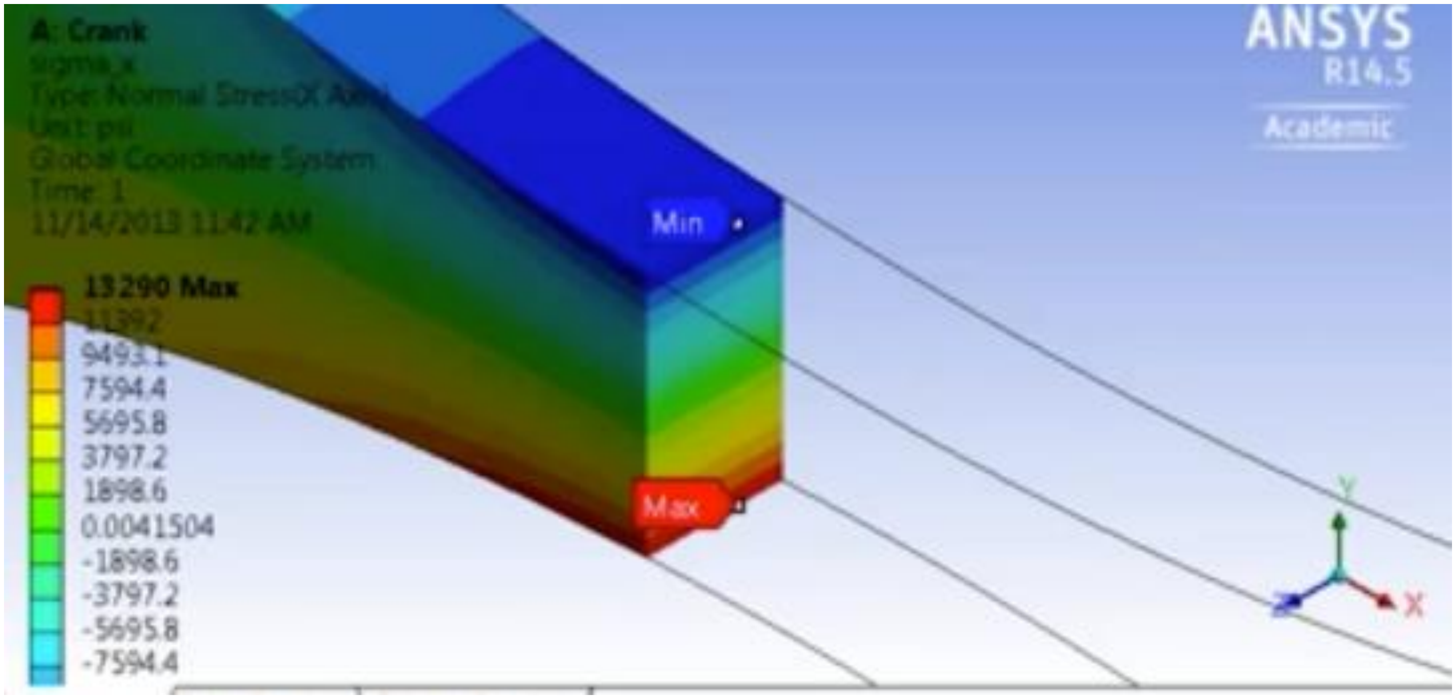
Bending Stress



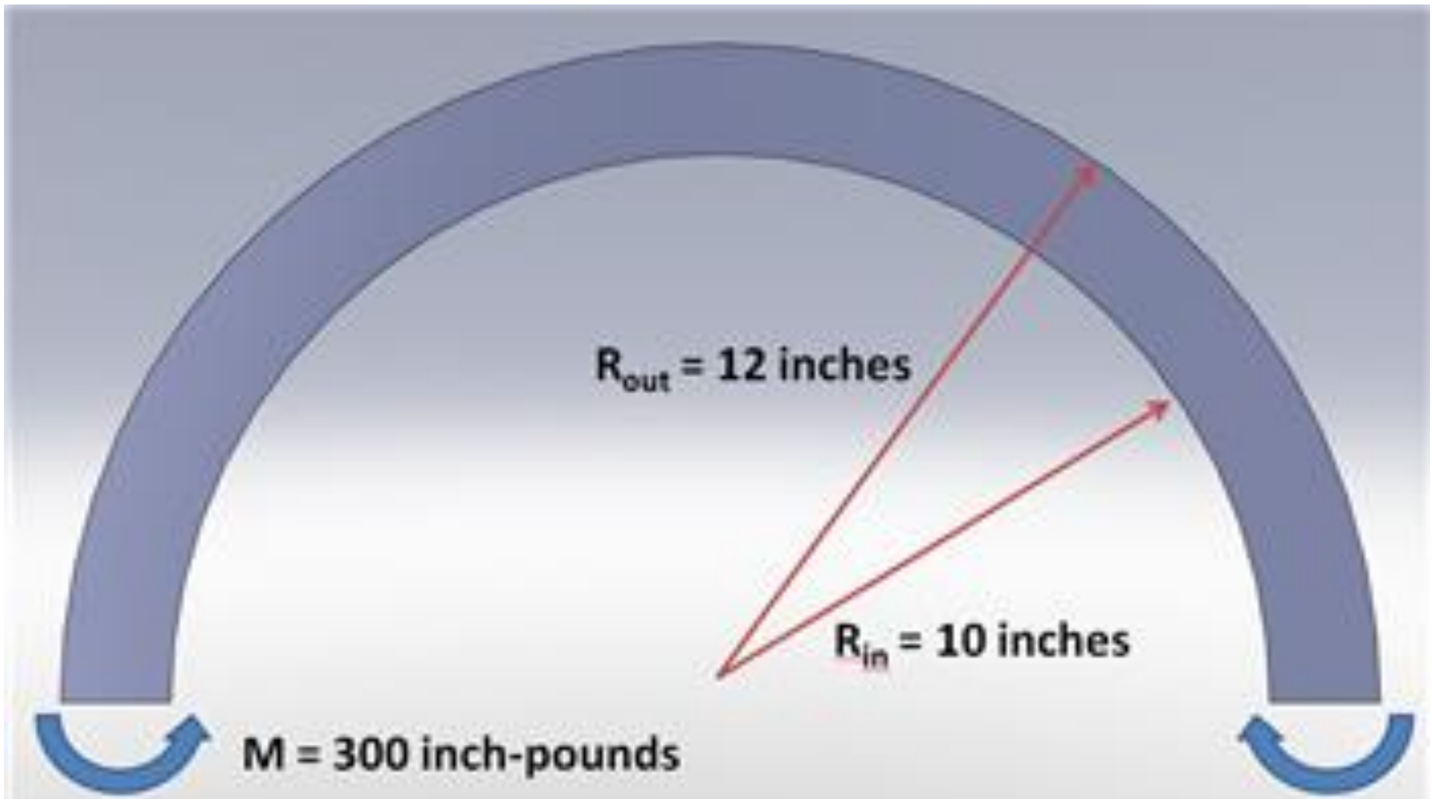
Bending Stress



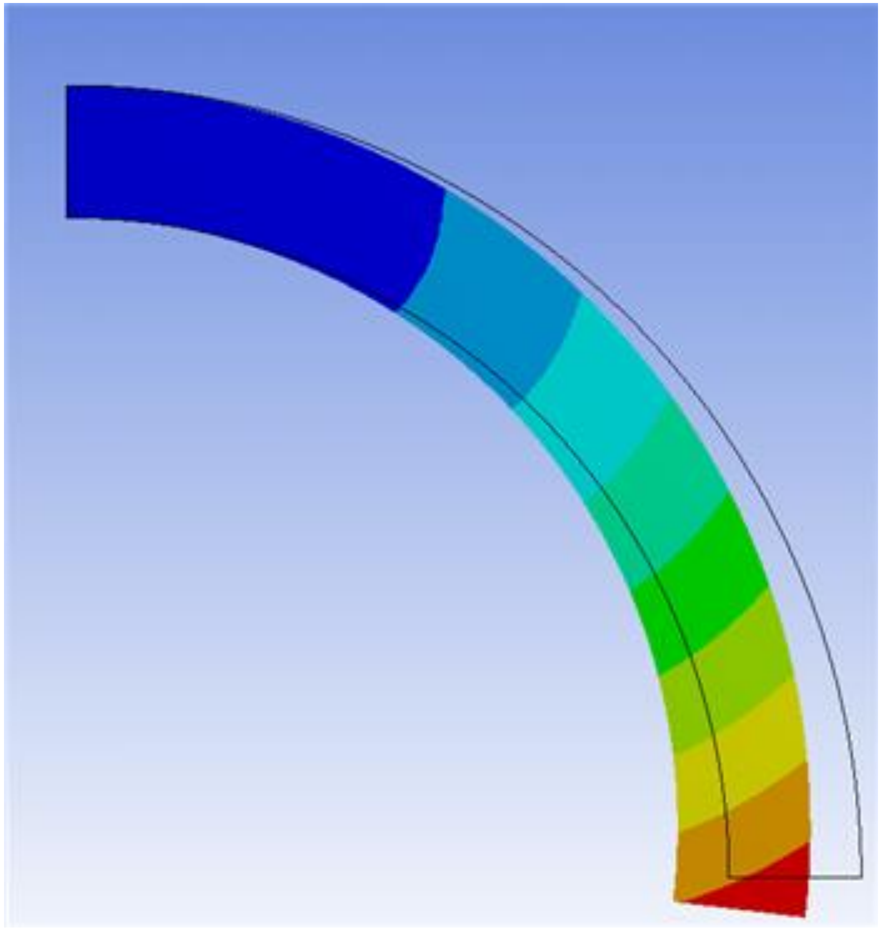
Bending Stress



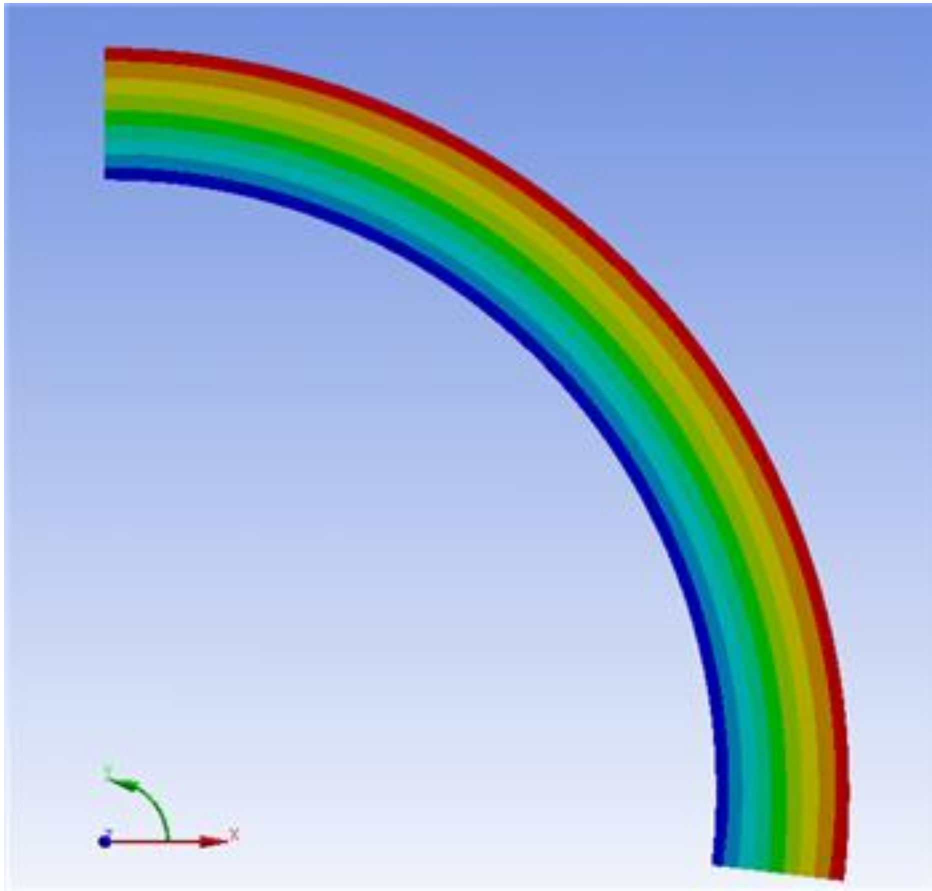
Bending Stress



Bending of Curved beam



Displacement

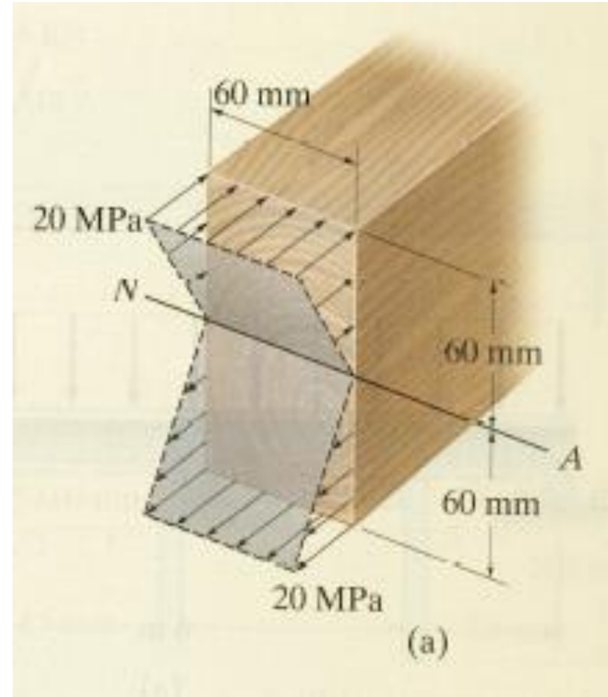


Stress in x direction

The Bending Formula

Example 1

A beam has a rectangular cross section and is subjected to the stress distribution shown in the figure. Determine the internal moment ***M*** at the section caused by the stress distribution.



$$I = \frac{bh^3}{12} = \frac{60(120)^3}{12} = 840*10^4 \text{ mm}^4$$

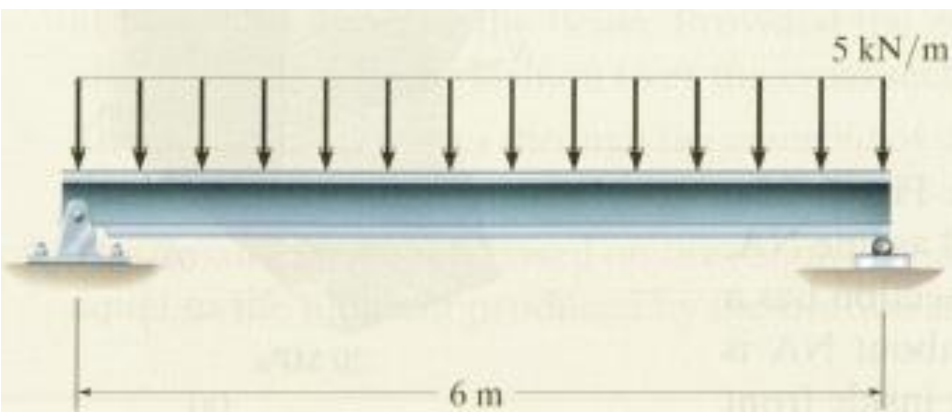
$$\sigma_{max} = \frac{M}{I}c \qquad \Rightarrow \qquad 20 = \frac{M(60)}{840*10^4}$$

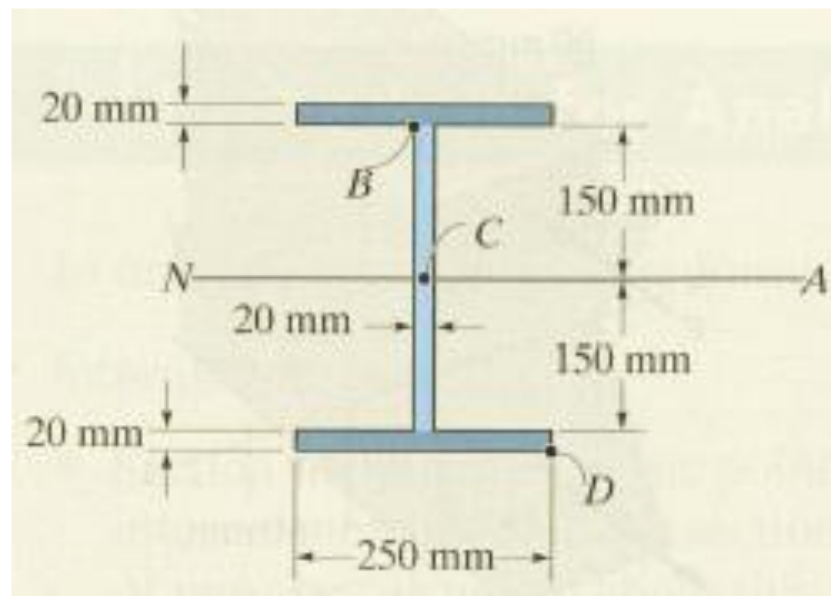
$$M = 288*10^4 \text{ N.mm}$$

The Bending Formula

Example 2

The simply supported beam has the cross-sectional area shown in the figure. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



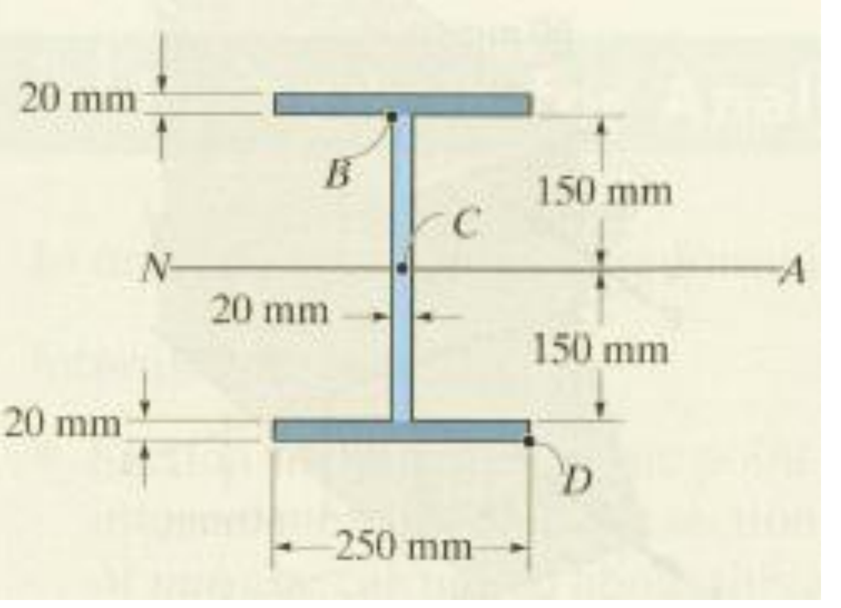


The Bending Formula

$$I = \sum \left(\bar{I} + Ad^2 \right)$$

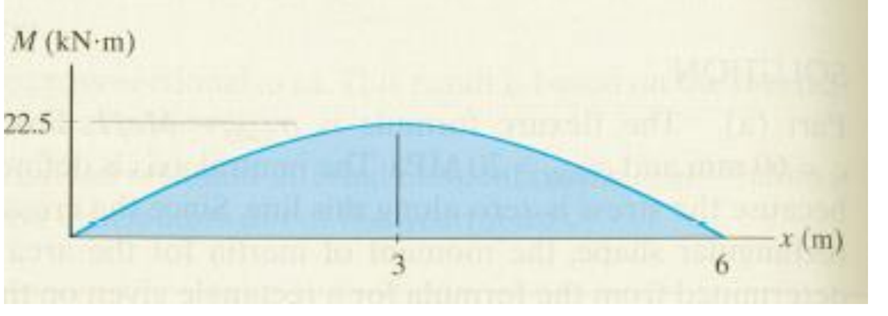
$$= 2\left[\frac{1}{12}(250)(20)^3 + 20*250*160^2 \right]$$

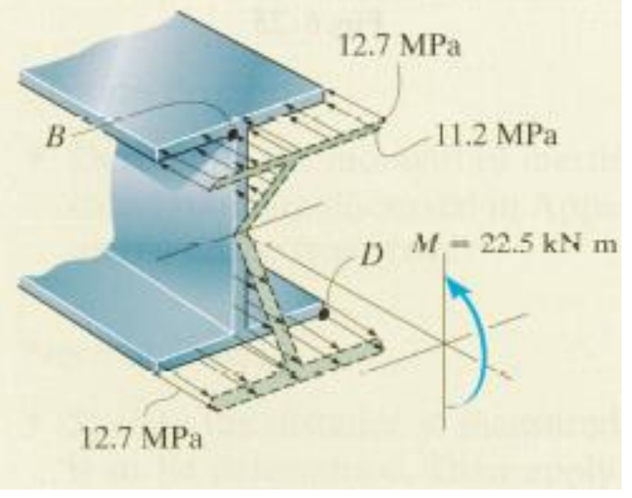
$$+ \left[\frac{1}{12}(20)(300)^3 \right] = 301.3*10^6 \text{ mm}^4$$



$$\sigma_{max} = \frac{M}{I}c;\ \sigma_{max} = \frac{22.5*10^6*170}{301.3*10^6}$$

$$= 12.7 \text{ MPa}$$

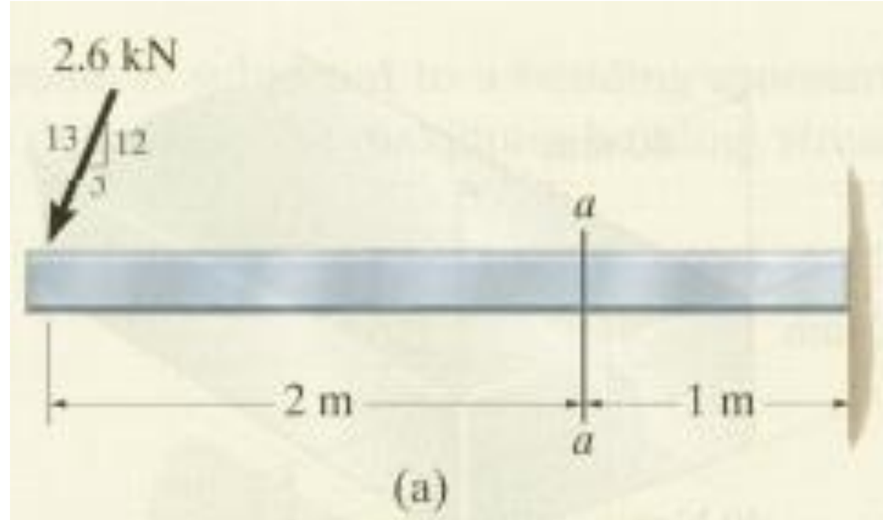


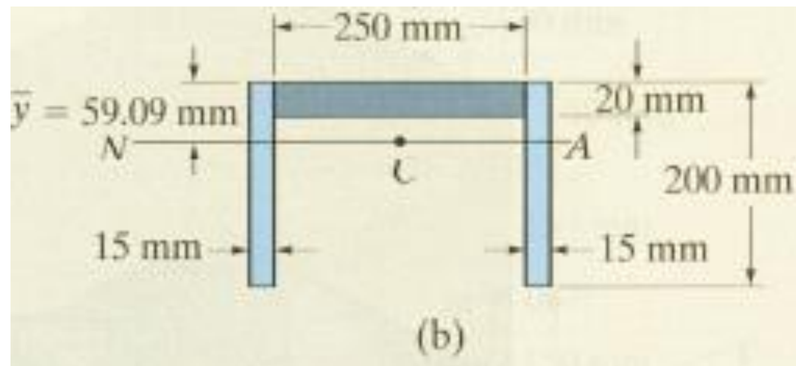


The Bending Formula

Example 3

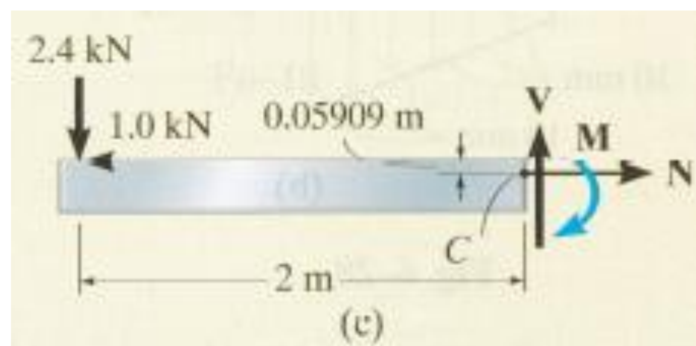
The beam shown in the figure has a cross-sectional area in the shape of a channel as shown in the attached figure. Determine the maximum bending stress that occurs in the beam at section *a-a*.





$$\bar{y} = \frac{\sum \tilde{y} A}{A} = \frac{2*100*15*200 + 10*20*250}{2*15*200 + 20*250}$$

$$= 59.09 \text{ mm}$$



$$\sum M_{NA} = 0$$

$$2400*2000 + 1000*59.09 - M = 0$$

$$M = 4859*10^3 \text{ N.mm}$$

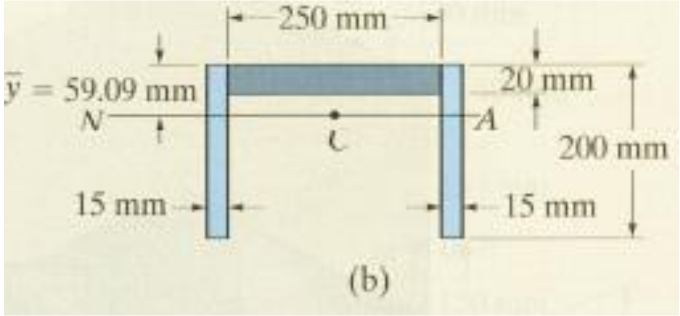
The Bending Formula

$$I = \sum \left(\bar{I} + Ad^2 \right)$$

$$= 2\left[\frac{1}{12}(15)(200)^3 + 15*200*(100-59.09)^2 \right]$$

$$+ \left[\frac{1}{12}(250)(20)^3 + 250*20*(59.09-10)^2 \right]$$

$$= 42.26*10^6 \text{ mm}^4$$

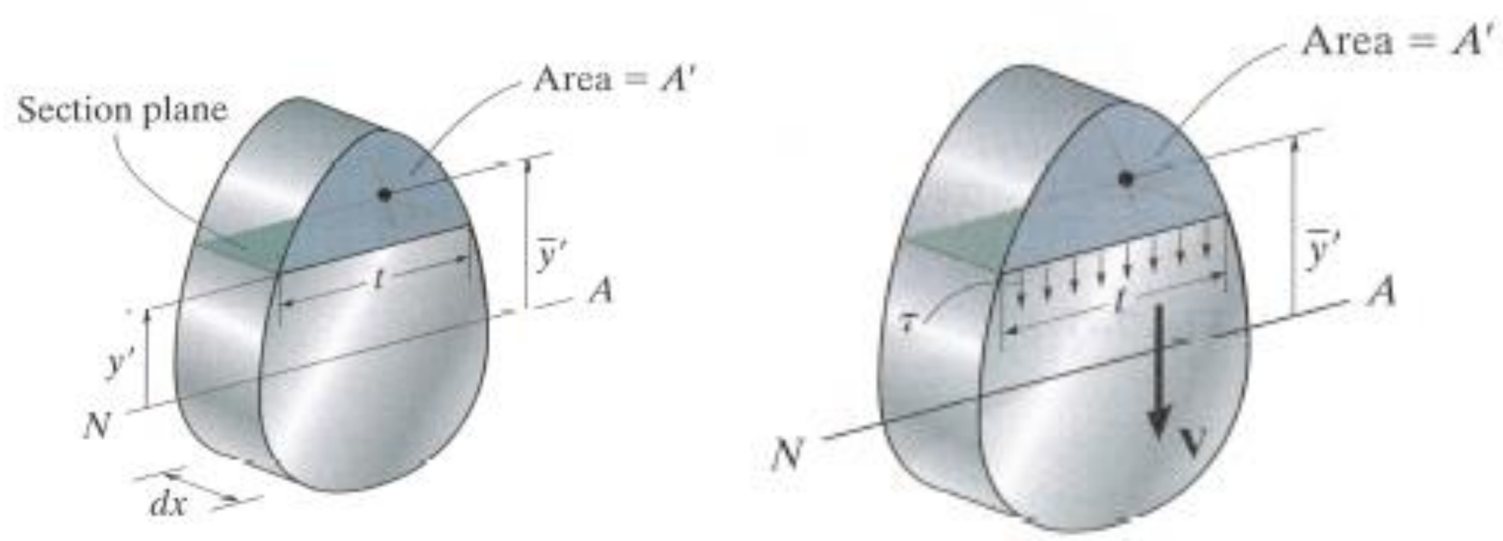


$$\sigma_{\max} = \frac{M}{I}c; \; \sigma_{\max} = \frac{4.859*10^6*(200-59.09)}{42.25*10^6}$$

$$= 16.2 \text{ MPa}$$

The Shear Formula

$$\tau = \frac{VQ}{It}$$



Where

τ The shear stress in the member at the point located a distance y' from the neutral axis. This stress is assumed to be constant and therefore averaged across the width t of the member.

V The internal resultant shear force, determined from the method of sections and the equations of equilibrium.

The Shear Formula

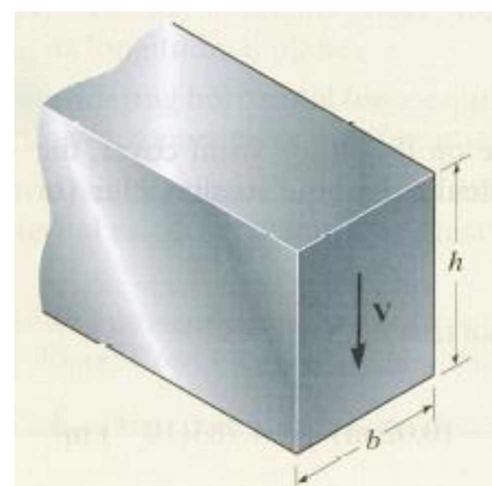
I The moment of inertia of the entire cross-sectional area calculated about the neutral axis.

t The width of the member's cross-sectional area, measured at the point where τ is to be calculated.

Q = $\bar{y}'A'$ where A' is the area of the top portion of the member's cross-sectional area, above the section plane where t is measured, and $\bar{y}'$ is the distance from the neutral axis to the centroid of A'.

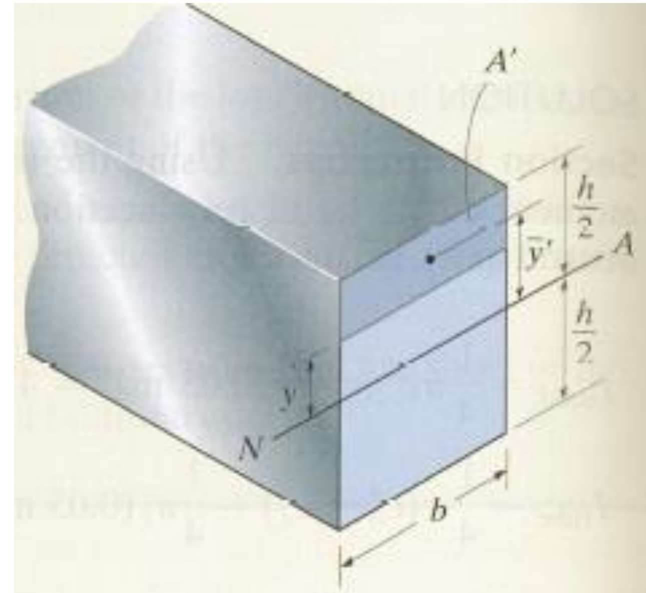
Example 1

Determine the distribution of the shear stress over the cross section of the beam shown in the attached figure.



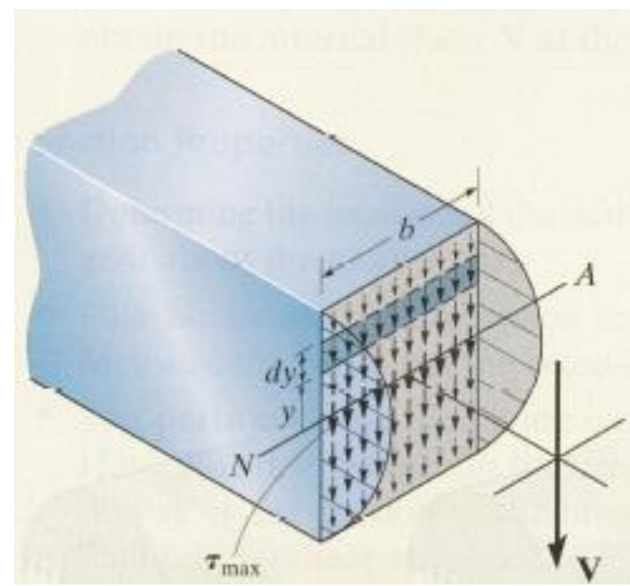
The Shear Formula

$$Q = \bar{y}'A' = \left(y + \frac{\frac{h}{2} - y}{2} \right)\left(\frac{h}{2} - y \right) b = \frac{1}{2}\left(\frac{h^2}{4} - y^2 \right)$$



Applying the shear formula, we have

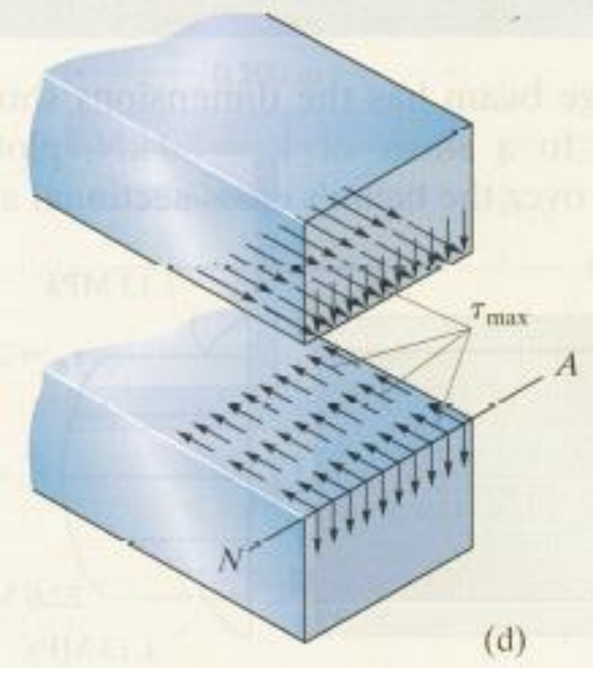
$$\tau = \frac{VQ}{It} = \frac{V\left(\frac{1}{2}\right)\left(\frac{h^2}{4} - y^2\right)b}{\left(\frac{1}{12}bh^3\right)b} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

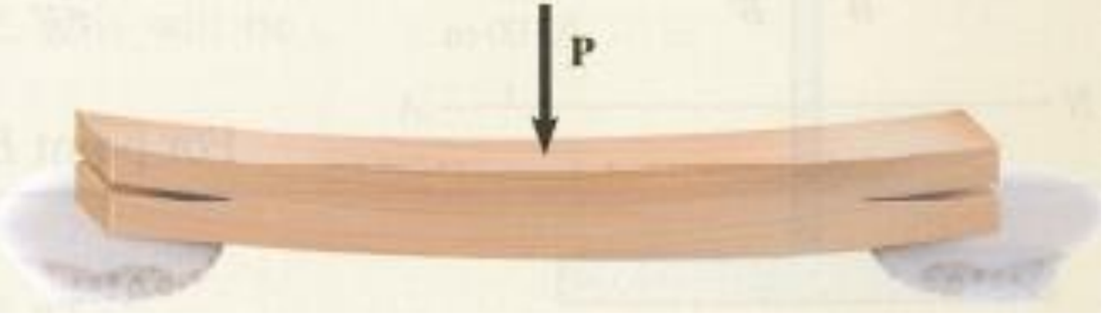


This indicates that the shear stress distribution over cross section is parabolic with maximum value of (1.5*V*/*A*) at the neutral axis and zero at top and bottom as shown in the attached figure.

The Shear Formula

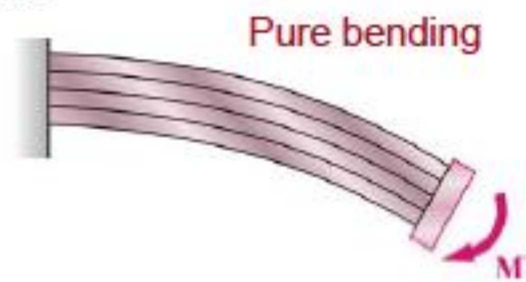
It is important to realize that τ_{max} also acts in the longitudinal direction of the beam as shown in the attached figure. It is this stress that can cause a timber beam to fail as shown in the figure. Here horizontal splitting of the wood starts to occur through the neutral axis at the beam's ends, since the vertical reactions subject the beam to large shear stress and wood has a low resistance to shear along its grains, which are oriented in the longitudinal direction.

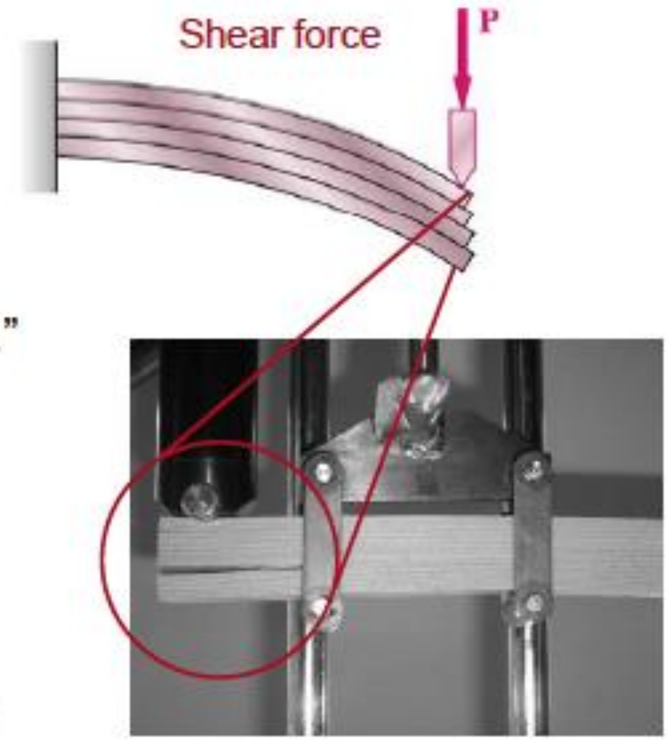




Shear Stress due to bending

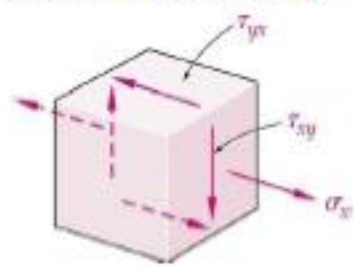
- Consider a cantilever beam composed of separate planks clamped at one end:





- Shear force causes tendency to "slide."

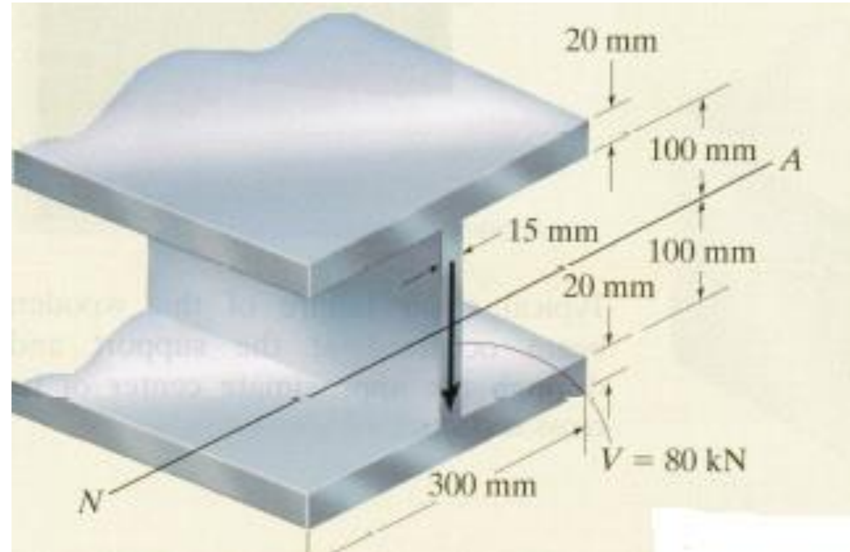
- Stresses are equal in horizontal and vertical directions.



The Shear Formula

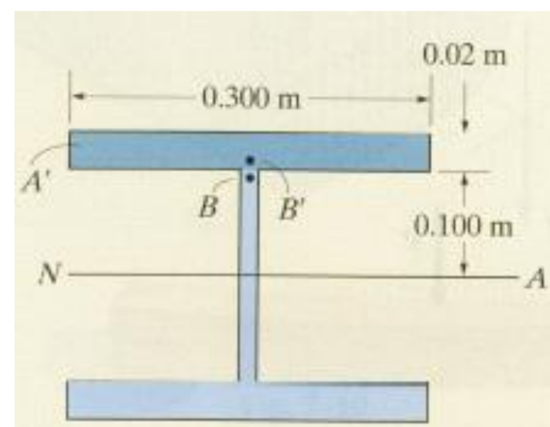
Example 2

A steel wide-flange beam has the dimensions shown in the figure. If is subjected to a shear of **V** = 80 kN, plot the shear-stress distribution acting over the beam's cross sectional area.



$$I = \left[\frac{15(200)^3}{12}\right] + 2\left[\frac{300(20)^3}{12} + 300(20)(110)^2\right] = 1556*10^5 \ \ mm^4$$

For point *B′*



$$t_{B'} = 300 \ \text{mm}$$

$$Q_{B'} = \bar{y}'A' = 110(300*20) = 66*10^4 \ \text{mm}^3$$

The Shear Formula

$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80*1000*66*10^4}{1556*10^5*300} = 1.13 \text{ MPa}$$

For point *B*

$$t_B = 15 \text{ mm}$$

$$Q_B = Q_{B'} = 66*10^4 \text{ mm}^3$$

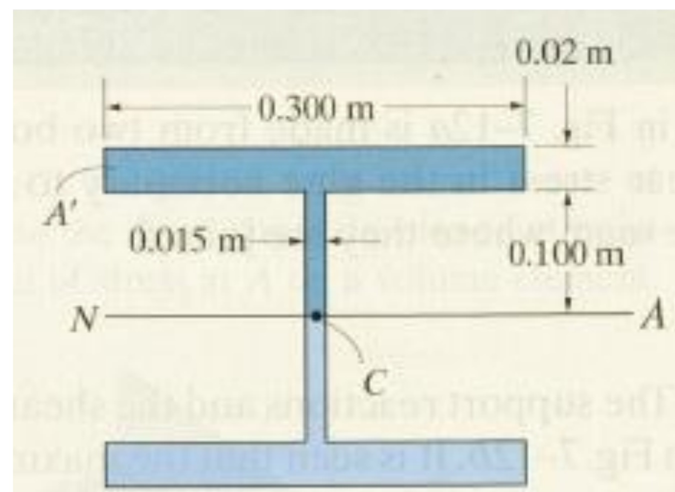
$$\tau_B = \frac{VQ_B}{It_B} = \frac{80*1000*66*10^4}{1556*10^5*15} = 22.6 \text{ MPa}$$

For ppint *C*

$$t_C = 15 \text{ mm}$$

$$Q_C = \sum \bar{y}'A' = 110(20*300) + 50(15*100)$$

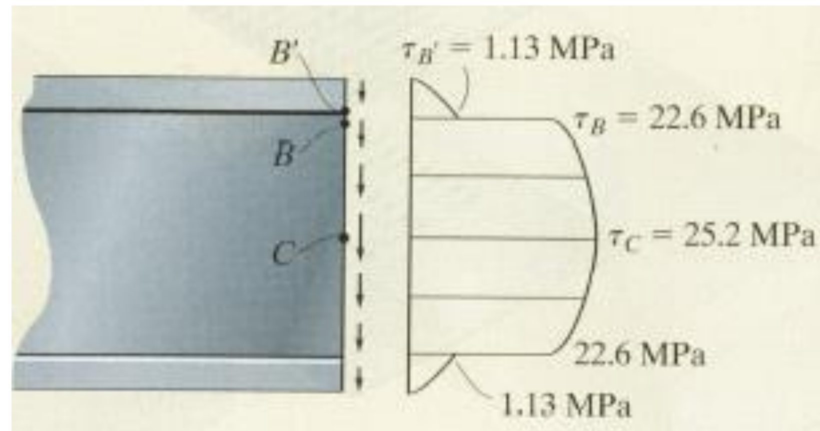
$$= 735*10^3 \text{ mm}^3$$



The Shear Formula

$$\tau_C = \frac{VQ_C}{It_C} = \frac{80*1000*735*10^3}{1556*10^5*15} = 25.2 \text{ MPa}$$

From the attached figure, note that the shear stress occurs in the web and is almost uniform throughout its depth, varying from 22.6 MPa to 25.2 MPa. It is for this reason that for design, some codes permit the use of calculating the average shear stress on the cross section of the web rather than using the shear formula; that is,

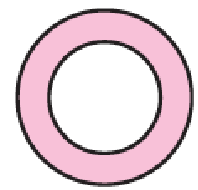
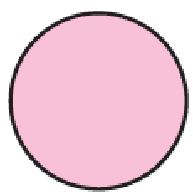
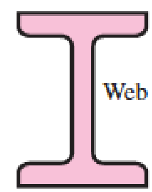


$$\tau_{avg} = \frac{V}{A} = \frac{80*1000}{15*200} = 26.7 \text{ MPa}$$

The Shear Formula

Table 3–2

Formulas for Maximum Shear Stress Due to Bending

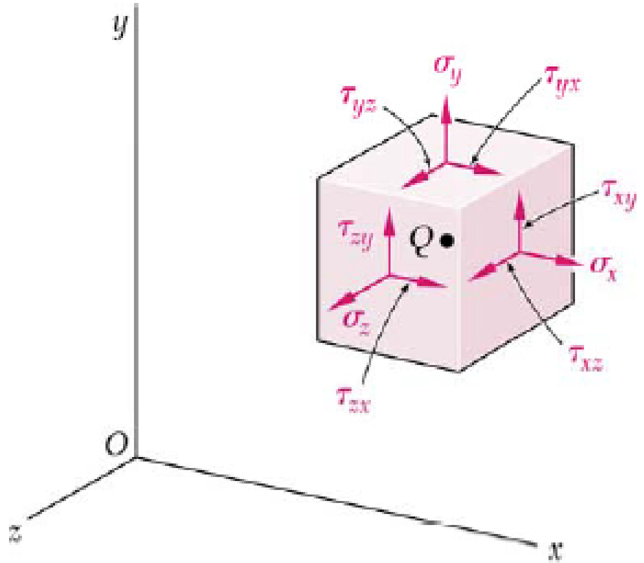
Beam Shape	Formula	Beam Shape	Formula
 Rectangular	$\tau_{max} = \frac{3V}{2A}$	 Hollow, thin-walled round	$\tau_{max} = \frac{2V}{A}$
 Circular	$\tau_{max} = \frac{4V}{3A}$	 Structural I beam (thin-walled)	$\tau_{max} = \frac{V}{A_{web}}$

State of Stress Caused by Combined Loadings

In previous chapters we developed methods for determining the stress distribution in a member subjected to either an internal axial force, a shear force, a bending moment, or a torsional moment. Most often, however, the cross section of a member is subjected to several of these loadings simultaneously. When this occurs, the method of superposition can be used to determine the resultant stress distribution.

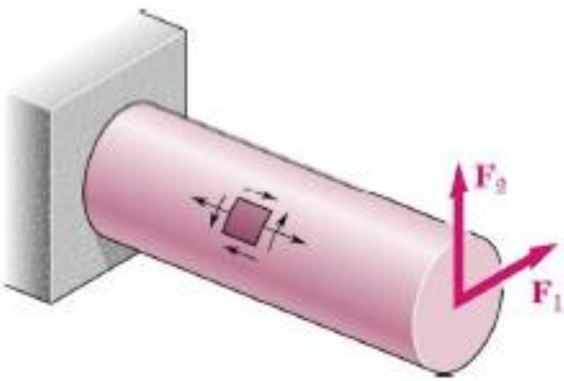
Transformation of Stress

- Recall the general state of stress at a point can be written in terms of 6 components: σ_x, σ_y, σ_z, $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$
- This general "stress state" is independent of the coordinate system used.
- The **components** of the stress state in the different directions **do** depend on the coordinate system.

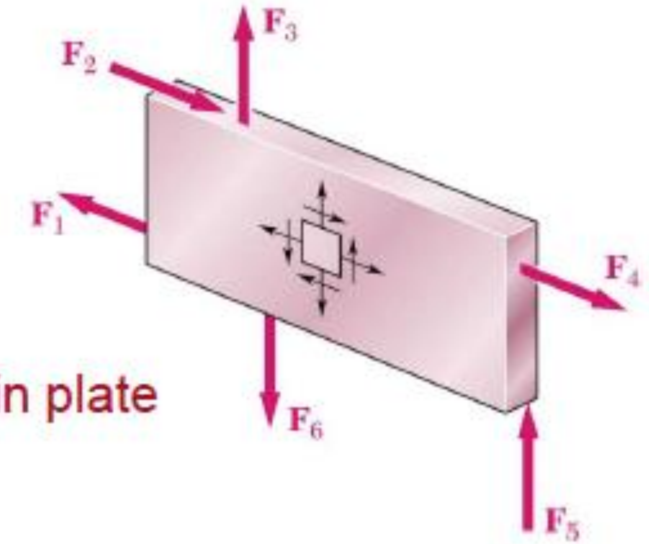


Transformation of Stress cont'd

- Consider a state of plane stress: $\sigma_z = \tau_{xz} = \tau_{yz} = 0$
- Where does this occur?



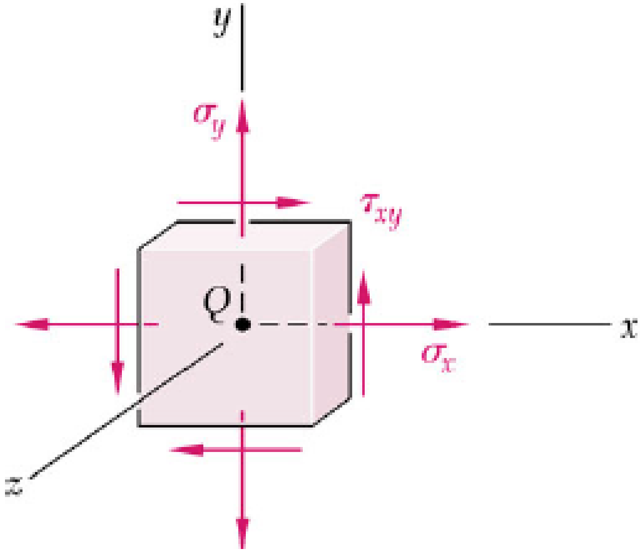
Outer surface



Thin plate

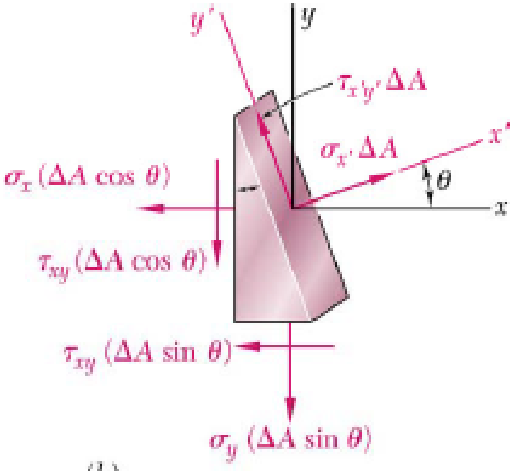
Transformation of Stress cont'd

- What do we want to calculate?
  - Principle stresses (σ maximum and σ minimum)
  - Principle planes of stresses (orientation at which they occur)
- Slice cube at an angle θ to the x axis (new coordinates x', y').
- Define forces in terms of angle and stresses.



Transformation of Stress cont'd

- Sum forces in x' direction.



$$\sigma_x'\Delta A = \sigma_x(\Delta A\cos\theta)\cos\theta + \sigma_y(\Delta A\sin\theta)\sin\theta$$
$$+\tau_{xy}(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\cos\theta)\sin\theta$$

$$\sigma_x' = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

- Sum forces in y' direction.

$$\tau_{xy}'\Delta A = -\sigma_x(\Delta A\cos\theta)\sin\theta + \sigma_y(\Delta A\sin\theta)\cos\theta - \tau_{xy}(\Delta A\sin\theta)\sin\theta + \tau_{xy}(\Delta A\cos\theta)\cos\theta$$

$$\tau_{xy}' = (\sigma_y - \sigma_x)\cos\theta\sin\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Transformation of Stress cont'd

$$\sigma_x' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\boxed{\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)}$$

To get σ_y', evaluate σ_x' at θ + 90º.

$$\boxed{\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)}$$

$$\tau_{xy}' = (\sigma_y - \sigma_x)\cos\theta\sin\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

$$\boxed{\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)}$$

Trig identities

$$2\sin\theta\cos\theta = \sin(2\theta) \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\cos^2\theta - \sin^2\theta = \cos(2\theta) \qquad \sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

Transformation of Stress cont'd

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y' = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}' = -\frac{\sigma_x-\sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

Now, let's perform some algebra:

$$\left(\sigma_x' - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{xy}^2{}' =$$

Constants (we can find these stresses).

$$\left(\sigma_x' - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{xy}^2{}' = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Variables

Principle and Max Shearing Stress

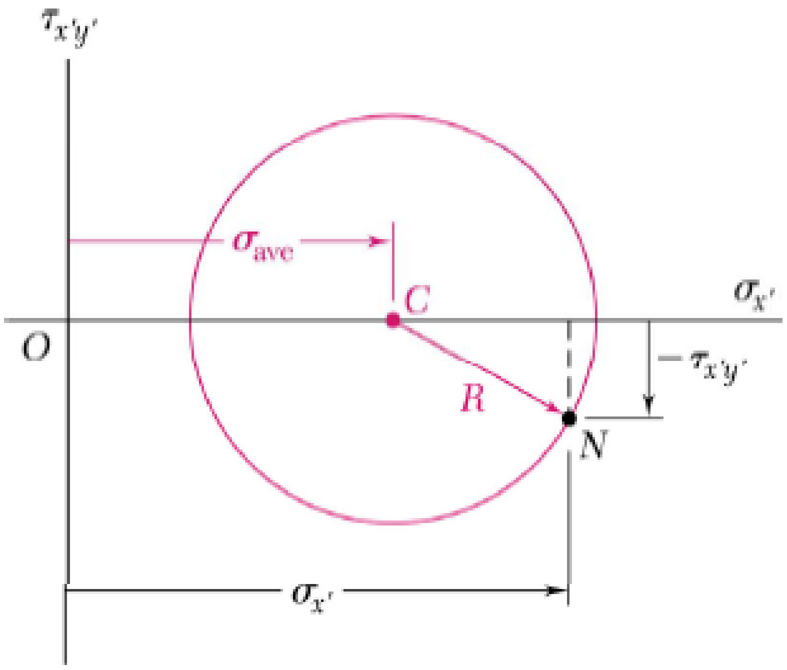
- Define

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) \qquad R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

- Plug into previous equation

$$\left(\sigma_x' - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2{}' = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

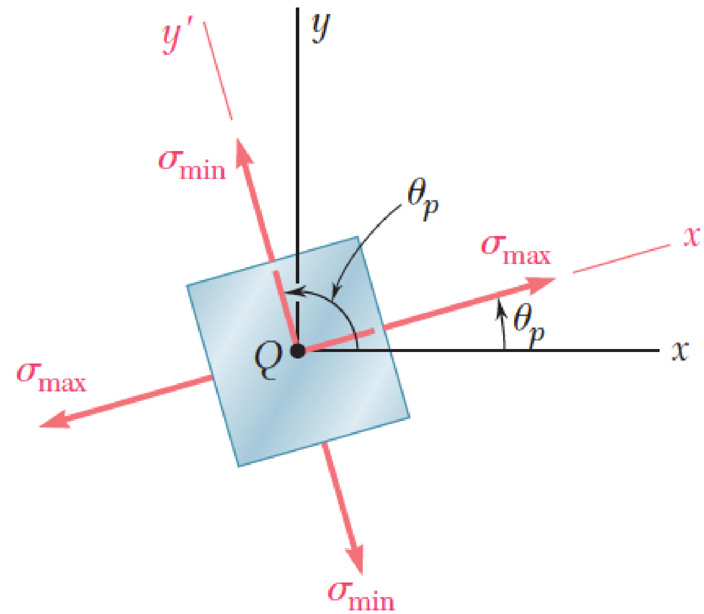
$$\boxed{(\sigma_{x1} - \sigma_{ave})^2 + \tau_{xy1}^2 = R^2}$$



- Which is the equation of a circle with center at $(\sigma_{ave}, 0)$ and radius R.

Summary

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



Principal Stresses

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Example 1

For the state of plane stress shown in Fig., determine (a) the principal planes, (b) the principal stresses, (c) the maximum shearing stress.

a. Principal Planes. Following the usual sign convention, the stress components are

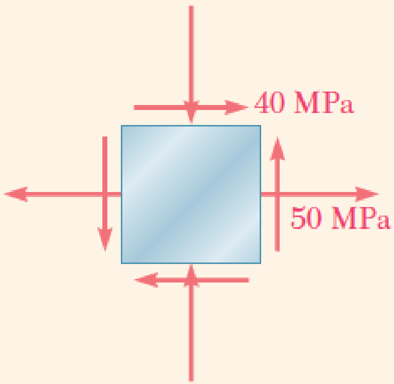
$$\sigma_x = +50\text{ MPa} \qquad \sigma_y = -10\text{ MPa} \qquad \tau_{xy} = +40\text{ MPa}$$

Substituting into Eq. (7.12),

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(+40)}{50-(-10)} = \frac{80}{60}$$

$$2\theta_p = 53.1^\circ \quad \text{and} \quad 180^\circ + 53.1^\circ = 233.1^\circ$$

$$\theta_p = 26.6^\circ \quad \text{and} \quad 116.6^\circ$$



(a)

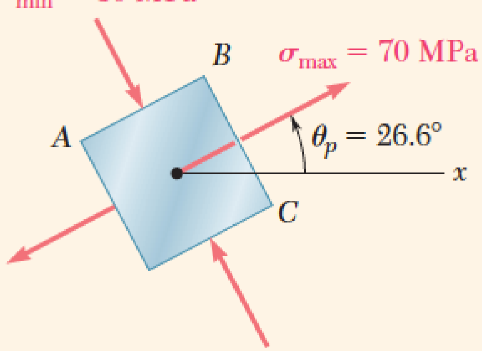
b. Principal Stresses. Equation (7.14) yields

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 20 \pm \sqrt{(30)^2 + (40)^2}$$

$$\sigma_{\max} = 20 + 50 = 70\text{ MPa}$$

$$\sigma_{\min} = 20 - 50 = -30\text{ MPa}$$



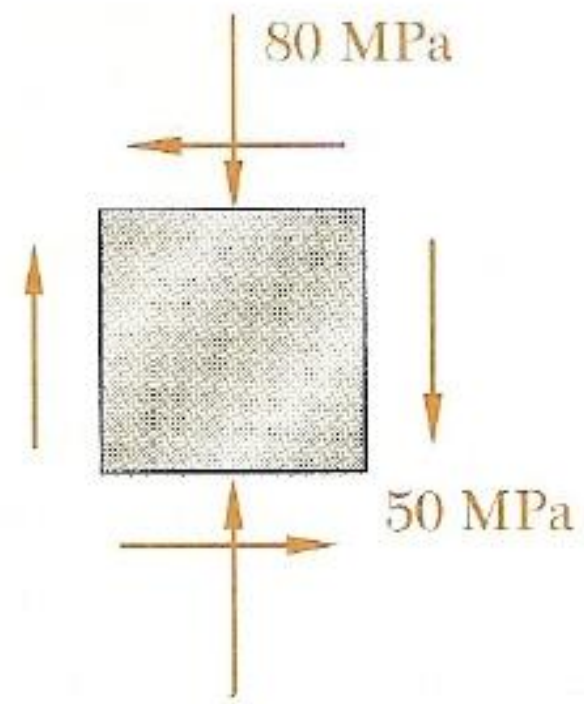
(b)

c. Maximum Shearing Stress. Equation (7.16) yields

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(30)^2 + (40)^2} = 50\,\text{MPa}$$

Example Problem 1

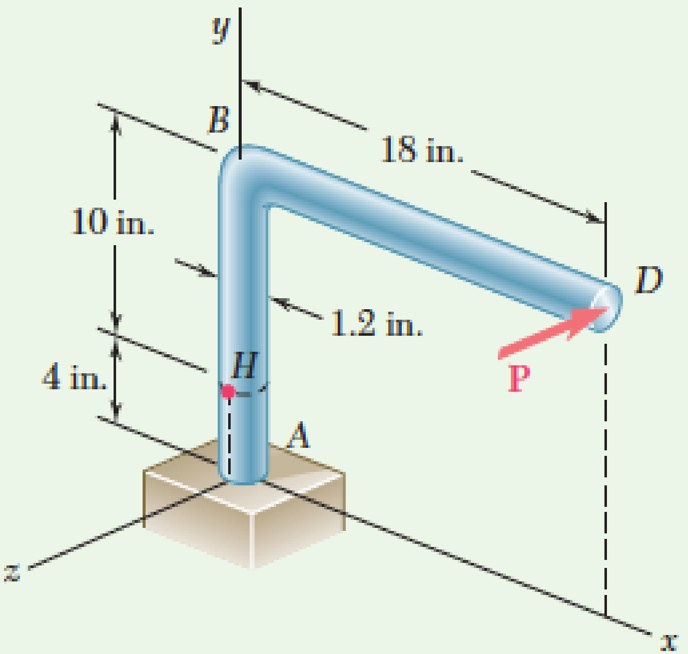
For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (a) 25° clockwise and (b) 10° counterclockwise.



Example Problem 2

Sample Problem 7.1 in Beer's Book

A single horizontal force **P** with a magnitude of 150 lb is applied to end *D* of lever *ABD*. Knowing that portion *AB* of the lever has a diameter of 1.2 in., determine (*a*) the normal and shearing stresses located at point *H* and having sides parallel to the *x* and *y* axes, (*b*) the principal planes and principal stresses at point *H*.



STRATEGY: You can begin by determining the forces and couples acting on the section containing the point of interest, and then use them to calculate the normal and shearing stresses acting at that point. These stresses can then be transformed to obtain the principal stresses and their orientation.

MODELING and ANALYSIS:

Force-Couple System. We replace the force **P** by an equivalent force-couple system at the center C of the transverse section containing point H (Fig.1):

$$P = 150 \text{ lb} \qquad T = (150 \text{ lb})(18 \text{ in.}) = 2.7 \text{ kip·in.}$$

$$M_x = (150 \text{ lb})(10 \text{ in.}) = 1.5 \text{ kip·in.}$$

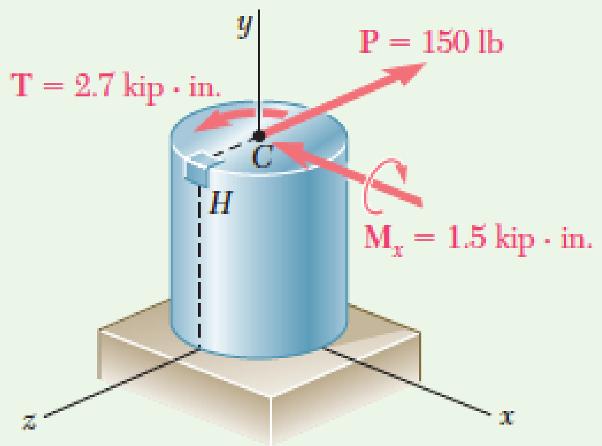


Fig. 1 Equivalent force-couple system acting on transverse section containing point *H*.

a. Stresses σ_x, σ_y, τ_{xy} at Point H. Using the sign convention shown in Fig. 7.2, the sense and the sign of each stress component are found by carefully examining the force-couple system at point *C* (Fig. 1):

$$\sigma_x = 0 \quad \sigma_y = +\frac{Mc}{I} = +\frac{(1.5 \text{ kip·in.})(0.6 \text{ in.})}{\frac{1}{4}\pi\,(0.6 \text{ in.})^4} \qquad \sigma_y = +8.84 \text{ ksi} \blacktriangleleft$$

$$\tau_{xy} = +\frac{Tc}{J} = +\frac{(2.7 \text{ kip·in.})(0.6 \text{ in.})}{\frac{1}{2}\pi\,(0.6 \text{ in.})^4} \qquad \tau_{xy} = +7.96 \text{ ksi} \blacktriangleleft$$

We note that the shearing force **P** does not cause any shearing stress at point *H*. The general plane stress element (Fig. 2) is completed to reflect these stress results (Fig. 3).

b. Principal Planes and Principal Stresses. Substituting the values of the stress components into Eq. (7.12), the orientation of the principal planes is

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(7.96)}{0 - 8.84} = -1.80$$

$$2\theta_p = -61.0° \qquad \text{and} \qquad 180° - 61.0° = +119°$$

$$\theta_p = -30.5° \qquad \text{and} \qquad +59.5° \blacktriangleleft$$

Substituting into Eq. (7.14), the magnitudes of the principal stresses are

$$\sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 8.84}{2} \pm \sqrt{\left(\frac{0 - 8.84}{2}\right)^2 + (7.96)^2} = +4.42 \pm 9.10$$

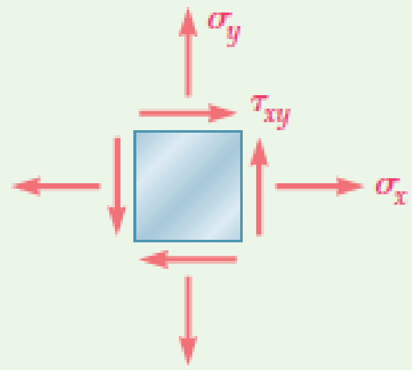


Fig. 2 General plane stress element (showing positive directions).

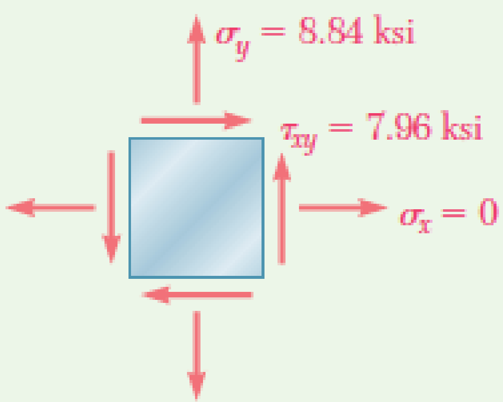


Fig. 3 Stress element at point *H*.

$$\sigma_{max} = +13.52 \text{ ksi}$$

$$\sigma_{min} = -4.68 \text{ ksi}$$

Considering face *ab* of the element shown, $\theta_p = -30.5°$ in Eq. (7.5) and $\sigma_{x'} = -4.68$ ksi. The principal stresses are as shown in Fig. 4.

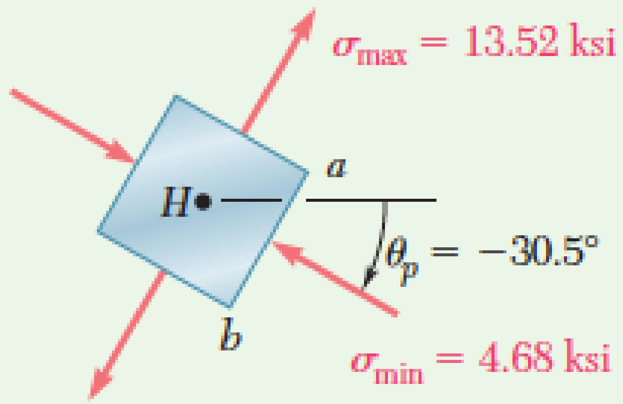
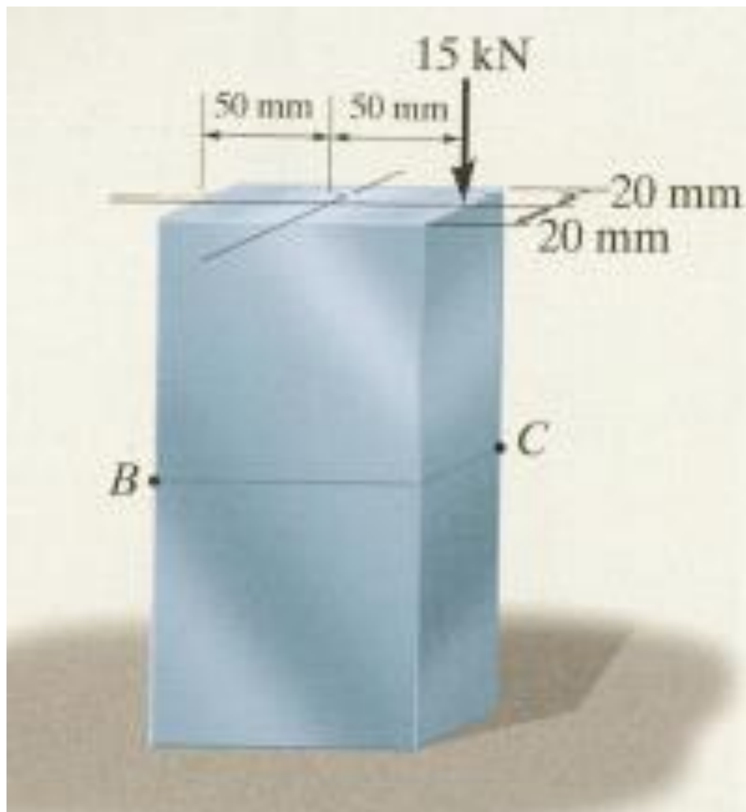


Fig. 4 Stress element at point *H* oriented in principal directions.

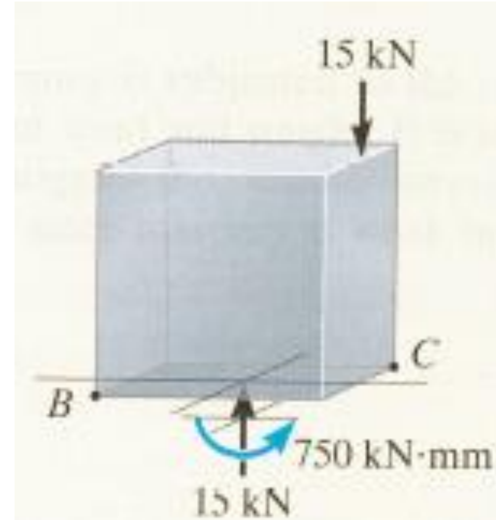
State of Stress Caused by Combined Loadings

Example 1

A force of 15 kN is applied to the edge of the member shown in the figure. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



State of Stress Caused by Combined Loadings



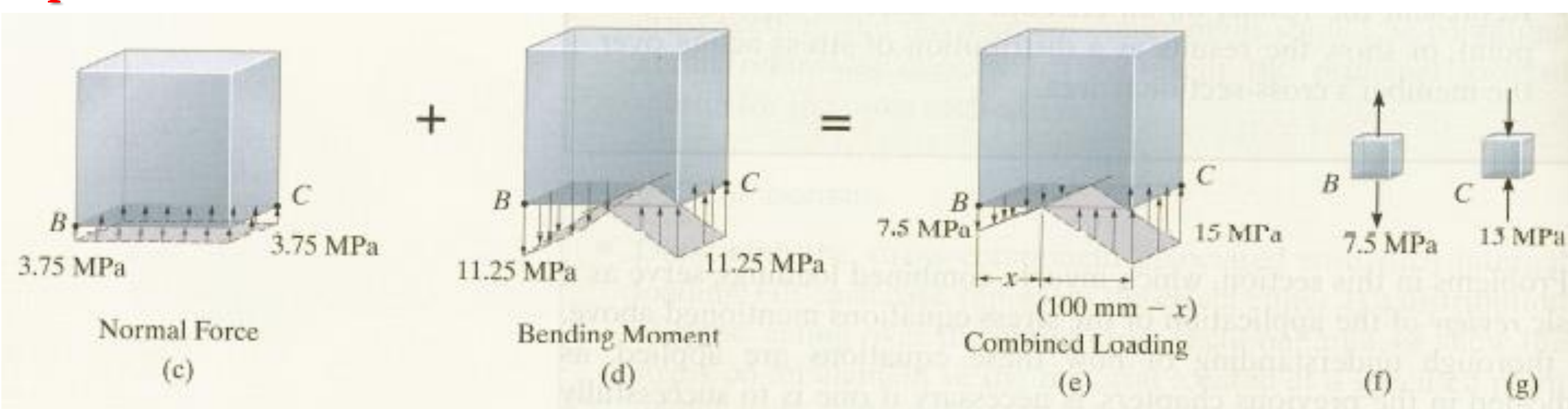
Normal Force

$$\sigma = \frac{P}{A} = \frac{15000}{40*100} = 3.75 \text{ MPa}$$

Bending Moment

$$\sigma_{\max} = \frac{Mc}{I} = \frac{15000*50}{\frac{1}{12}40(100)^3} = 11.25 \text{ MPa}$$

Superposition



State of Stress Caused by Combined Loadings

$$\frac{7.5}{x} = \frac{15}{100 - x}$$

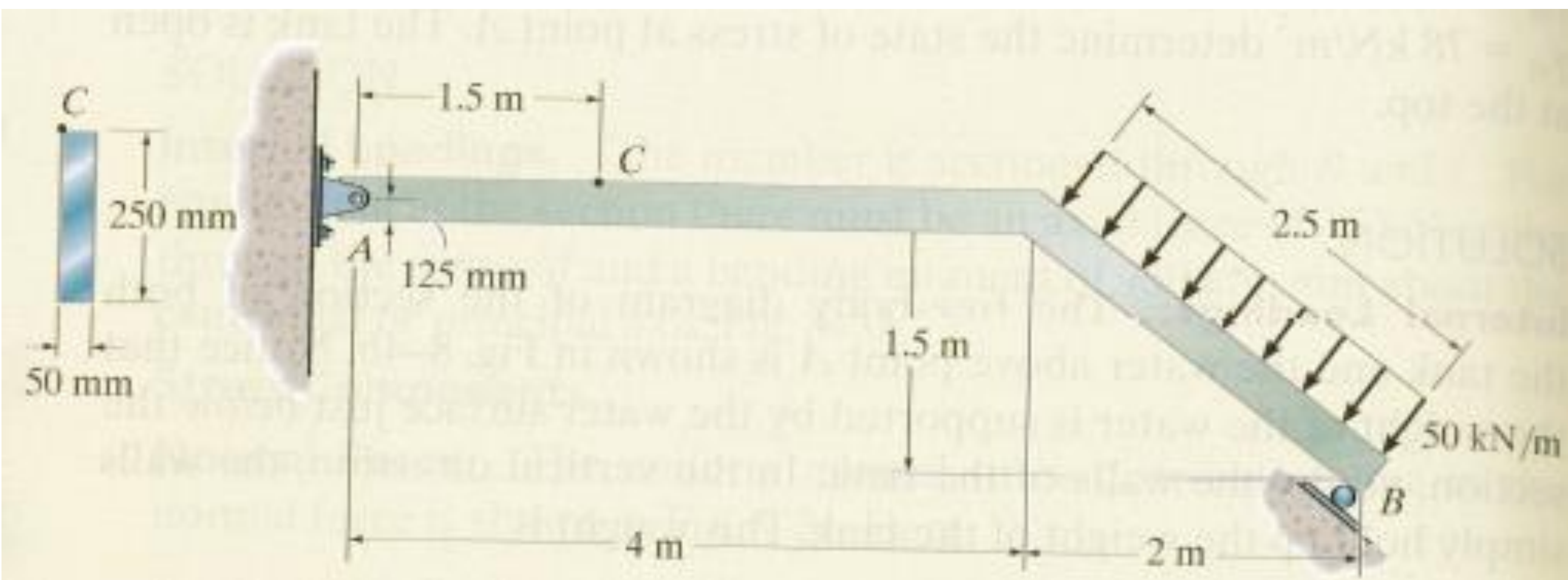
$$x = 33.3 \text{ mm}$$

$$\sigma_B = 11.25 - 3.75 = 7.5 \text{ MPa}$$

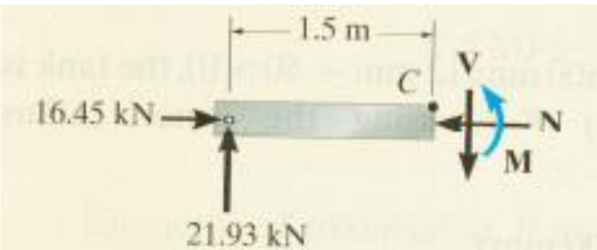
$$\sigma_C = -11.25 - 3.75 = -15 \text{ MPa}$$

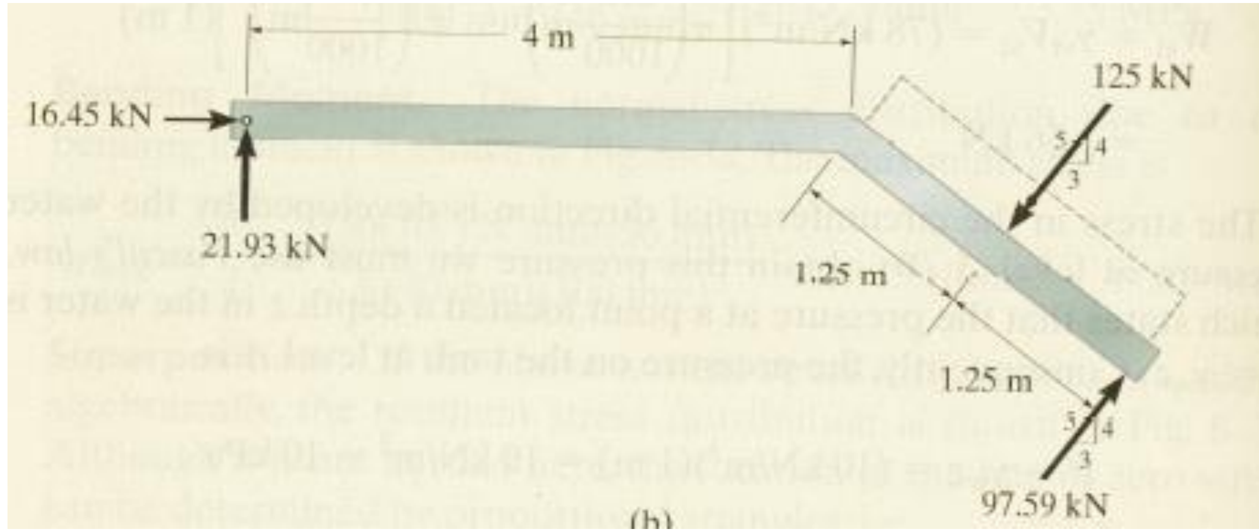
Example 2

The member shown in the figure has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.



State of Stress Caused by Combined Loadings





Normal Force

$$\sigma_c = \frac{P}{A} = \frac{16.45*1000}{50*250} = 1.32 \text{ MPa}$$

Shear Force

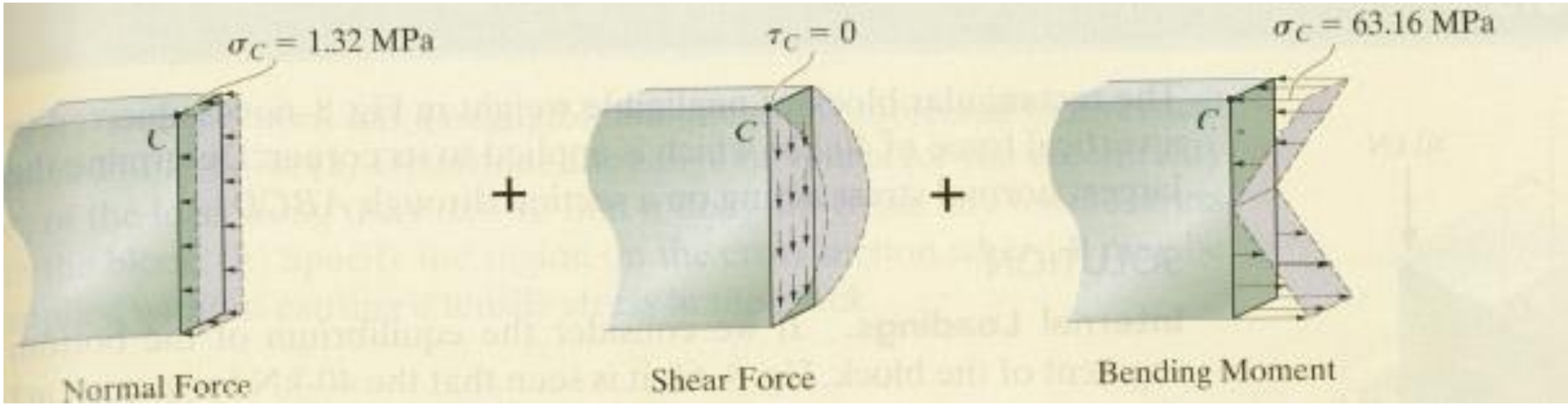
$$\tau_c = 0$$

State of Stress Caused by Combined Loadings

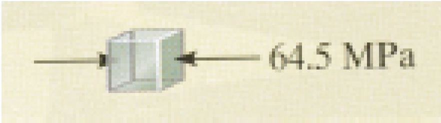
Bending Moment

$$\sigma_C = \frac{Mc}{I} = \frac{21.93*1000*1500*125}{\frac{1}{12}50(250)^3} = 63.16 \text{ MPa}$$

Superposition



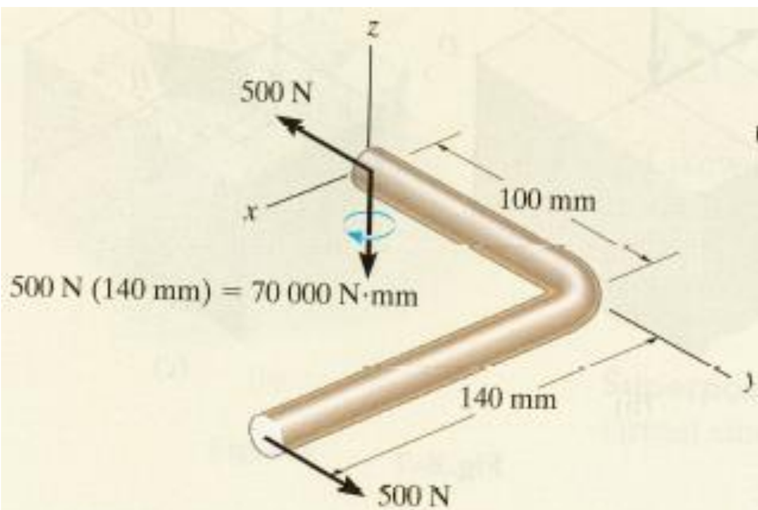
$$\sigma_C = -1.32 - 63.16 = -64.5 \text{ MPa}$$

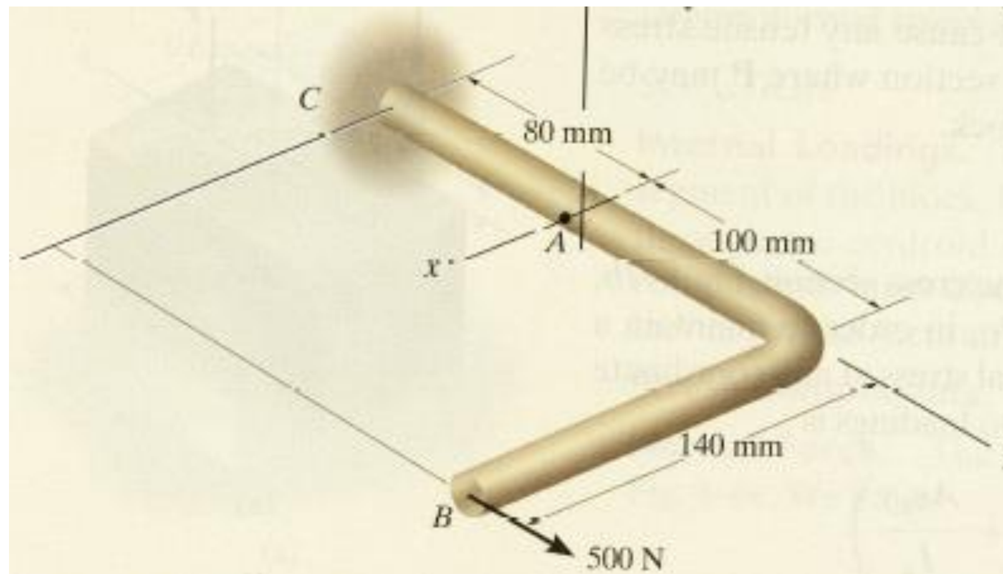


State of Stress Caused by Combined Loadings

Example 4

The solid rod shown in the figure has a radius of 7.5 mm. If it is subjected to the force 500 N, determine the state of stress at point *A*.





Normal Force

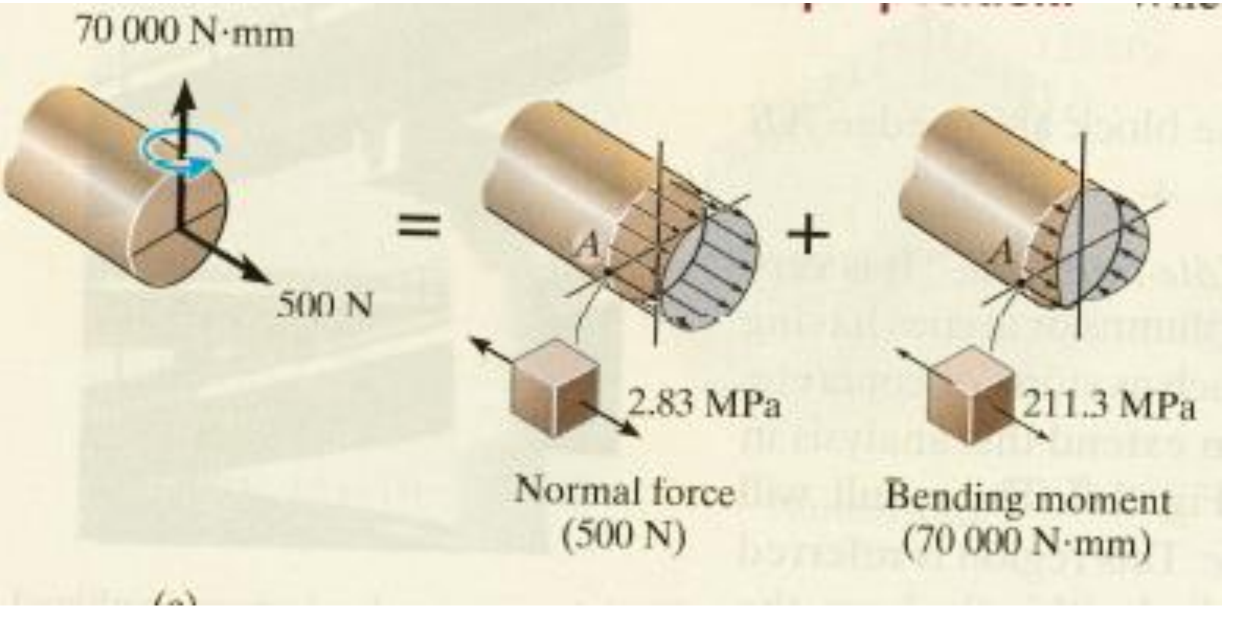
$$(\sigma_A)_y = \frac{P}{A} = \frac{500}{\pi(7.5)^2} = 2.83\ \text{MPa}$$

State of Stress Caused by Combined Loadings

Bending Moment

$$(\sigma_A)_y = \frac{Mc}{I} = \frac{500*140*7.5}{\frac{\pi}{64}(15)^4} = 211.3 \text{ MPa}$$

Superposition

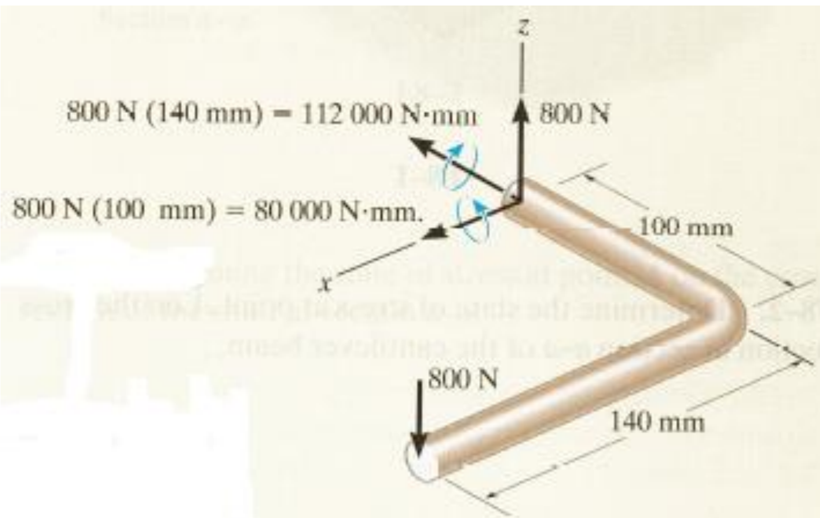


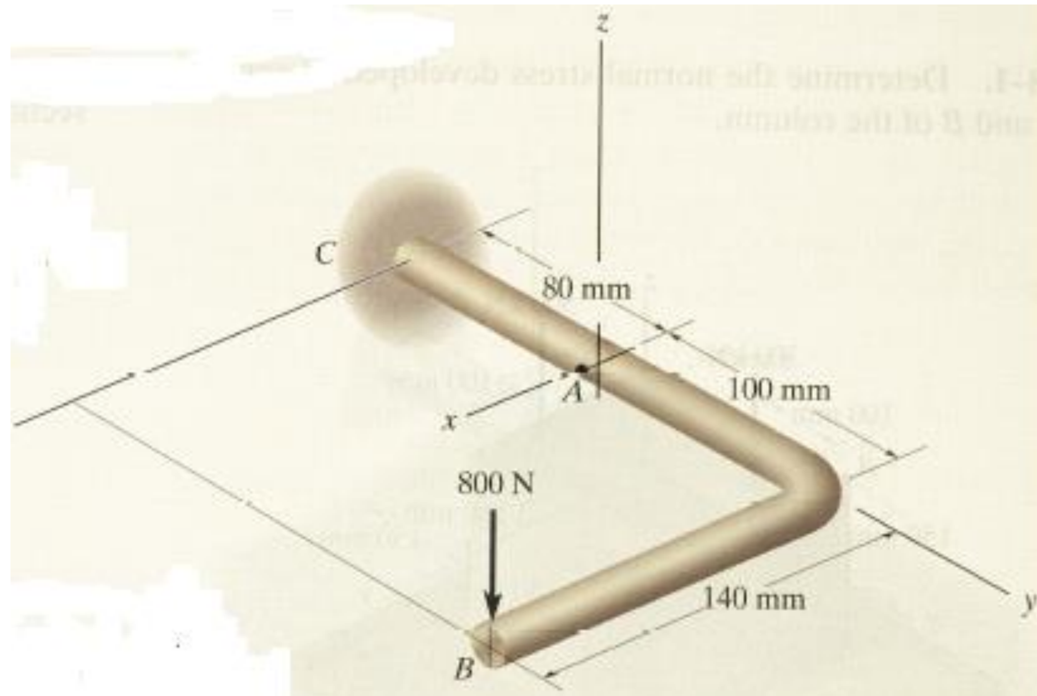
$$(\sigma_A)_y = 2.83 + 211.3 = 214.13 \text{ MPa}$$

State of Stress Caused by Combined Loadings

Example 5

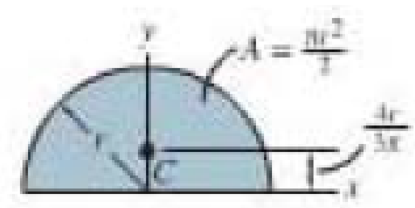
The solid rod shown in the figure has a radius of 7.5 mm. If it is subjected to the force 800 N, determine the state of stress at point *A*.





Shear Force

$$Q = \bar{y}'A' = \frac{4*7.5}{3\pi}\left(\frac{1}{2}\pi(7.5)^2\right) = 281.25\ \text{mm}^3$$



State of Stress Caused by Combined Loadings

$$\left(\tau_{yz}\right)_A = \frac{VQ}{It} = \frac{800*281.25}{\left(\frac{\pi}{64}15^4\right)\left(2*7.5\right)} = 6.04 \text{ MPa}$$

Bending Moment

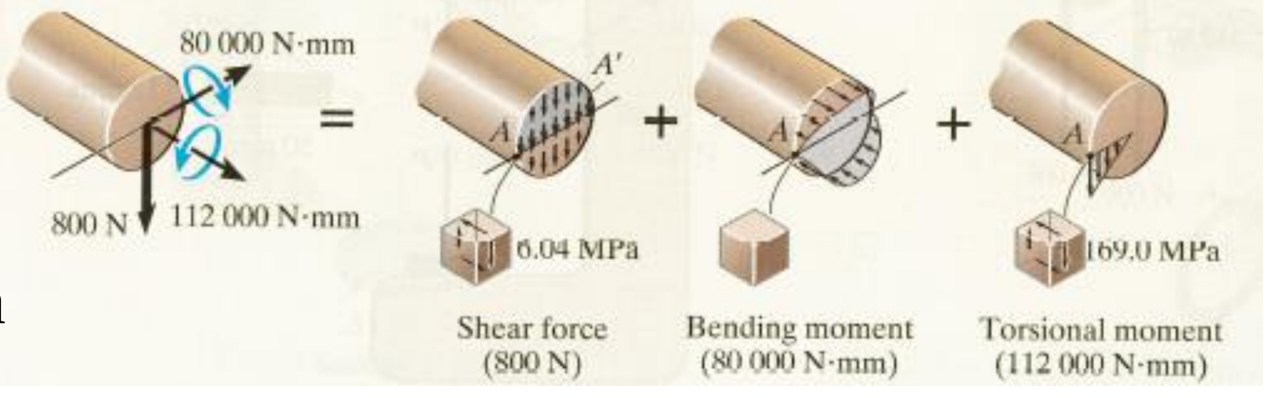
$$\sigma_A = 0$$

Torque

$$\left(\tau_{yz}\right)_A = \frac{Tc}{J} = \frac{112*1000*7.5}{\frac{\pi}{32}\left(15\right)^4} = 169.01 \text{ MPa}$$

Superposition

$$\left(\tau_{yz}\right)_A = 6.04 + 169.01$$
$$= 175.05 \text{ MPa}$$



State of Stress Caused by Combined Loadings

$$\left(\tau_{yz}\right)_A = \frac{VQ}{It} = \frac{800 * 281.25}{\left(\frac{\pi}{64}15^4\right)(2*7.5)} = 6.04 \text{ MPa}$$

Bending Moment

$$\sigma_A = 0$$

Torque

$$\left(\tau_{yz}\right)_A = \frac{Tc}{J} = \frac{112*1000*7.5}{\frac{\pi}{32}(15)^4} = 169.01 \text{ MPa}$$

Superposition

$$\left(\tau_{yz}\right)_A = 6.04 + 169.01$$
$$= 175.05 \text{ MPa}$$