

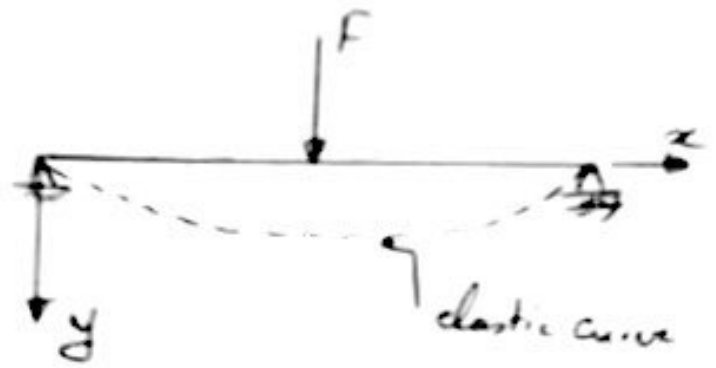
# Deflection

## Double integration method

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

this is the equation of

the elastic curve



$$\frac{dy}{dx} = f_1(x) + C_1$$

$$y = f_2(x) + C_1x + C_2$$

where  $C_1$  &  $C_2$  are constants and can be obtained from boundary conditions

## Boundary Conditions



simple support

$$y = 0.0$$

$$\frac{dy}{dx} \neq 0.0$$



fixed support

$$\frac{dy}{dx} = 0.0$$

$$y = 0.0$$

Then

$$y = \frac{1}{EI} \left[ \frac{W a^2}{4} x^2 + \frac{W}{24} x^3 - \frac{W a^3}{6} x^3 \right]$$

(2)  $a \leq x \leq a+b$

$$M_x = 0 \dots$$

$$EI \frac{d^2 y}{dx^2} = 0 \dots$$

$$EI \frac{dy}{dx} = C_3$$

$$EI y = C_3 x + C_4$$

Common boundary conditions

$$\text{at } x = a \quad \hookrightarrow \left. \frac{dy}{dx} \right|_1 = \left. \frac{dy}{dx} \right|_2$$

$$\& \quad \hookrightarrow y|_1 = y|_2$$

Then

$$C_3 = \checkmark$$

$$\& C_4 = \checkmark$$

$$EI \frac{dy}{dx} = -\frac{WL}{4} x^2 + \frac{W}{6} x^3 + \frac{WL^3}{24}$$

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$$EI y = -\frac{WL}{12} x^3 + \frac{W}{24} x^4 + \frac{WL^3}{24} x + C_2$$

2<sup>nd</sup> boundary condition

at  $x = 0.0 \rightarrow y = 0.0$

$$\boxed{0.0 = C_2}$$

Then

$$y = \frac{1}{EI} \left[ \frac{-WLx^3}{12} + \frac{Wx^4}{24} + \frac{WL^3}{24} x \right]$$

but  $y_{\max}$  at  $x = \frac{L}{2}$

$$\therefore y_{\max} = \frac{1}{EI} \left[ \frac{-WL \left(\frac{L}{2}\right)^3}{12} + \frac{W \left(\frac{L}{2}\right)^4}{24} + \frac{WL^3}{24} \cdot \frac{L}{2} \right]$$

$$\boxed{y_{\max} = \frac{5}{384} \frac{WL^4}{EI}}$$

$$M_x = Fx - F(x - 125) = 125F$$

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = -\frac{125F}{EI}$$

$$\frac{dy}{dx} = -\frac{125F}{EI}x + C_3$$

$$y = -\frac{125F}{2EI}x^2 + C_3x + C_4$$

boundary Conditions

$$\text{at } x = 250 \rightarrow \frac{dy}{dx} = 0.0$$

$$0.0 = -125 \frac{F}{EI} \times 250 + C_3$$

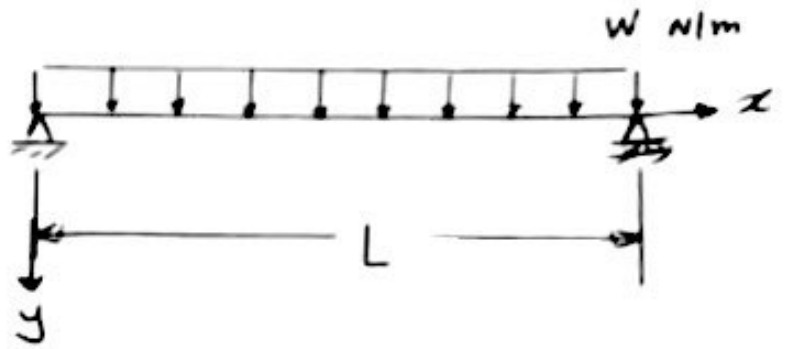
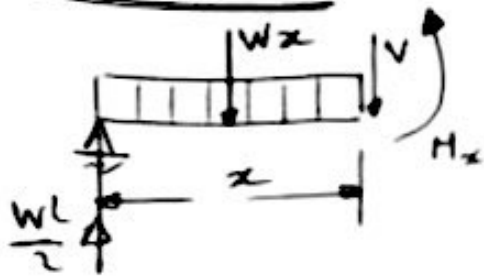
$$\boxed{C_3 = 2 \times (125)^2 \frac{F}{EI}}$$

$$\therefore y = -\frac{125F}{2EI}x^2 + \frac{2(125)^2}{EI}Fx + C_4 \rightarrow \textcircled{2}$$

Common boundary Condition

$$\text{at } x = 125 \quad \left. \frac{dy}{dx} \right|_1 = \left. \frac{dy}{dx} \right|_2$$

$$-\frac{F(125)^2}{2EI} + C_1 = -\frac{125F}{EI} \times 125 + 2 \times (125)^2 \frac{F}{EI}$$

Example 2

$$\frac{WL}{2} x - Wx \frac{x}{2} - M_x = 0.0$$

$$M_x = \frac{WL}{2} x - W \frac{x^2}{2}$$

$$\frac{d^2y}{dx^2} = - \frac{M_x}{EI}$$

$$EI \frac{d^2y}{dx^2} = - \frac{WL}{2} x + W \frac{x^2}{2}$$

$$EI \frac{dy}{dx} = - \frac{WL}{4} x^2 + \frac{W}{6} x^3 + C_1$$

1<sup>st</sup> boundary condition

$$\text{at } x = \frac{L}{2} \rightarrow \frac{dy}{dx} = 0.0$$

$$\therefore 0.0 = - \frac{WL}{4} \frac{L^2}{4} + \frac{W}{6} \frac{L^3}{8} + C_1$$

$$C_1 = \frac{WL^3}{24}$$

$$\therefore C_1 = \frac{3}{2} \frac{(125)^2 F}{EI}$$

at  $x = 125$

$$y_1 = y_2$$

$$-\frac{F(125)^3}{6EI} + \frac{3}{2} \frac{(125)^2 F}{EI} = -\frac{(125)^3 F}{2EI} + \frac{2(125)^3 F}{EI} + C_4$$

$$C_4 = -\frac{(125)^3 F}{6EI}$$

$$y_{\max} = y_2 \text{ at } x = 250$$

$$\therefore y_{\max} = -\frac{125F}{2EI} (250)^2 + \frac{2(125)^2 F}{EI} (250) - \frac{1}{6} \frac{(125)^3 F}{EI}$$

$$y_{\max} = \frac{11(125)^3 F}{6EI}$$

### Example 3

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$$(1) \quad 0.0 \leq x \leq a$$

$$M_x = -W(a-x) \left( \frac{a-x}{2} \right)$$

$$EI \frac{d^2 y}{dx^2} = -M_x$$

$$= W(a-x) \left( \frac{a-x}{2} \right) = \frac{Wa^2}{2} - Wax + \frac{Wx^2}{2}$$

$$EI \frac{dy}{dx} = \frac{Wa^2}{2} x + \frac{Wx^3}{6} - \frac{Wa}{2} x^2 + C_1$$

but at  $x=0.0 \rightarrow \frac{dy}{dx} = 0.0$

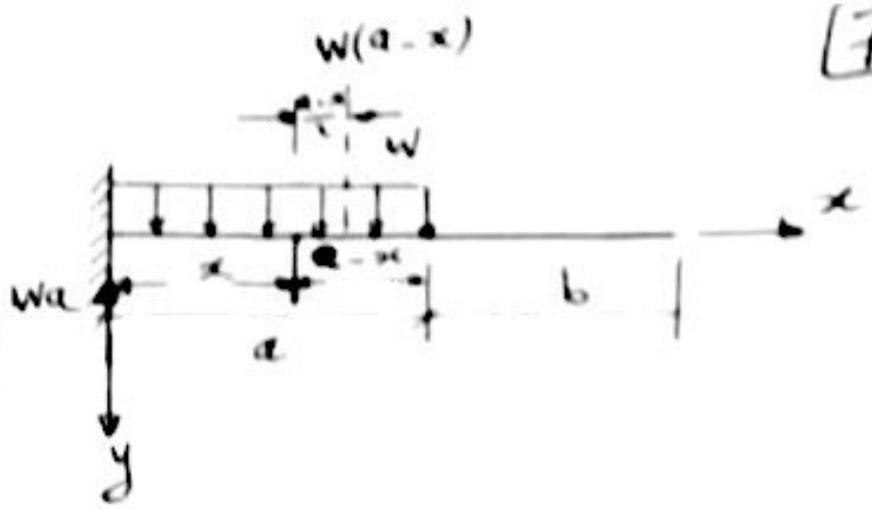
$$\boxed{0.0 = C_1}$$

$$\therefore EI \frac{dy}{dx} = \frac{Wa^2}{2} x + \frac{Wx^3}{6} - \frac{Wa}{2} x^2$$

$$EI y = \frac{Wa^2}{4} x^2 + \frac{Wx^4}{24} - \frac{Wa}{6} x^3 + C_2$$

but at  $x=0.0 \rightarrow y = 0.0$

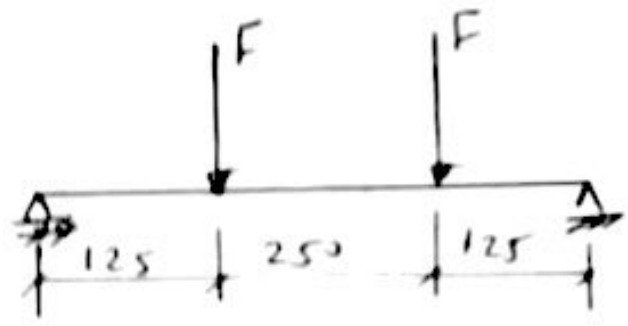
$$\boxed{C_2 = 0.0}$$



Example 1

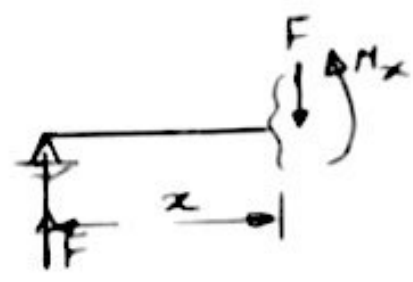
Req  $y_{max}$

sol



(1) for  $0.0 \leq x \leq 125$

$$M_x = Fx$$



$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = -\frac{Fx}{EI}$$

$$\frac{dy}{dx} = -\frac{Fx^2}{2EI} + C_1$$

$$y = -\frac{Fx^3}{6EI} + C_1x + C_2$$

Boundary Conditions

at  $x = 0.0$        $y = 0.0$        $\rightarrow$   $C_2 = 0.0$

Then

$$y = -\frac{Fx^3}{6EI} + C_1x \rightarrow \textcircled{1}$$

(2) for  $125 \leq x \leq 250$

