**Matlab Sheet 2 Hints**

**Single Degree of Freedom - Free and Forced undamped system**

## Problem 2

Use the stiffness of the wing only case in the calculations

e) write the whole terms of the Fourier series in your program

## Matlab Sheet 4 Hints

## Two Degree of Freedom Systems

Problem 1

Givens

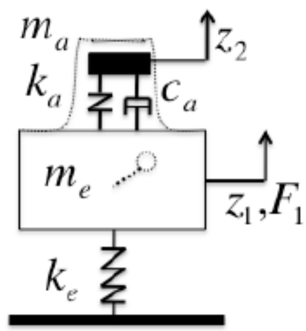
$k_e = 4.934802 \times 10^5$ N/m, $m_e = 500$ Kg, $\omega = 1500$ RPM= 157.08 rad/sec.

$F_{un} = m_e \Omega^2 \sin(\omega t) = 0.02 \times 500 \times 9.8 \times \sin(\omega t) = 98 \ \sin(\omega t) = F_0 \sin(\omega t)$

$F_0 = 98$

$X_2 = 1$ *inch= 0.0254 m*

### a) For undamped system ($c_a = c_e = zero$)




$$\begin{bmatrix} m_e & 0 \\ 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_e + k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_o \ \sin \omega t \\ 0 \end{bmatrix}$$

The steady-state solution, $x_j (t) = X_j \sin \omega t$ $\qquad$ $j = 1,2$

$$\begin{bmatrix} (-\omega^2 m_e + k_e + k_a) & -k_a \\ -k_a & (-\omega^2 m_a + k_a) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (-\omega^2 m_e + k_e + k_a) & -k_a \\ -k_a & (-\omega^2 m_a + k_a) \end{bmatrix}^{-1} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} (k_a - \omega^2 m_a) & k_a \\ k_a & (k_e + k_a - \omega^2 m_e) \end{bmatrix}}{(k_e + k_a - \omega^2 m_e)(k_a - \omega^2 m_a) - k_a^2} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$X_1 = \frac{(k_a - \omega^2 m_a) \times F_0}{(k_e + k_a - \omega^2 m_e)(k_a - \omega^2 m_a) - k_a^2}$$

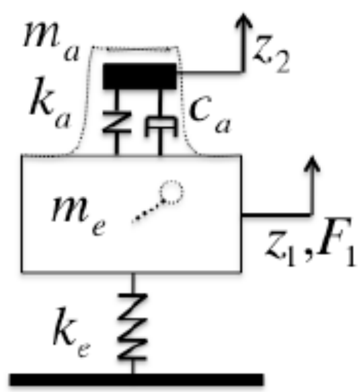
$$X_2 = \frac{k_a \times F_0}{(k_e + k_a - \omega^2 m_e)(k_a - \omega^2 m_a) - k_a^2}$$

$$F_{Tr} = k_e \times X_1 = \frac{k_e \times (k_a - \omega^2 m_a) \times F_0}{(k_e + k_a - \omega^2 m_e)(k_a - \omega^2 m_a) - k_a^2}$$

If $\omega^2 = \frac{k_a}{m_a}$

$$X_1 = zero \;, \qquad X_2 = \frac{F_0}{k_a} \;, \qquad F_{Tr} = k_e \times X_1 = zero$$

**For Damped System ($c_e = zero$)**




$$\begin{bmatrix} m_e & 0 \\ 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_a & -c_a \\ -c_a & c_a \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_e + k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_o \sin \omega t \\ 0 \end{bmatrix}$$

The steady-state solution, $x_j(t) = X_j \sin \omega t$ $\qquad j = 1,2$

$$\begin{bmatrix} (-\omega^2 m_e + \omega c_a + k_e + k_a) & -\omega c_a - k_a \\ -\omega c_a - k_a & (-\omega^2 m_a + \omega c_a + k_a) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (-\omega^2 m_e + \omega c_a + k_e + k_a) & -\omega c_a - k_a \\ -\omega c_a - k_a & (-\omega^2 m_a + \omega c_a + k_a) \end{bmatrix}^{-1} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} (-\omega^2 m_a + \omega c_a + k_a) & \omega c_a + k_a \\ \omega c_a + k_a & (-\omega^2 m_e + \omega c_a + k_e + k_a) \end{bmatrix}}{D} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$D = (-\omega^2 m_e + \omega c_a + k_e + k_a)(-\omega^2 m_a + \omega c_a + k_a) - (\omega c_a + k_a)^2$$

$$X_1 = \frac{(-\omega^2 m_a + \omega c_a + k_a)\times F_0}{D}$$

$$X_2 = \frac{(\omega c_a + k_a)\times F_0}{D}$$

$$F_{Tr} = k_e \times X_1 = \frac{k_e \times (-\omega^2 m_a + \omega c_a + k_a)\times F_0}{D}$$

You can use Eq 5.37 in Inman book (4th edition) shown below:

$$\frac{X}{\Delta} = \frac{Xk}{F_0} = \sqrt{\frac{(2\zeta r)^2 + (r^2 - \beta^2)^2}{(2\zeta r)^2(r^2 - 1 + \mu r^2)^2 + [\mu r^2\beta^2 - (r^2 - 1)(r^2 - \beta^2)]^2}} \quad (5.37)$$

d) Read Section 5.4 Damping in Vibration Absorption in Inman book ($4^{th}$ edition)

e) use Matlab function (eig) to calculate the mode shapes and compare it with hand calculations.

Problem 2

Find the equations of motion of the two degree of freedom system shown in the figure, then plot the time response.