

**Sheet 6**  
**Two Degree of Freedom Modal Analysis**

1. Consider the system of Figure 1 consisting of two pendulums coupled by a spring. Determine the natural frequency and mode shapes. Plot the mode shapes as well as the solution to an initial condition consisting of the first mode shape for  $k = 20 \text{ N/m}$ ,  $l = 0.5 \text{ m}$  and  $m_1 = m_2 = 10 \text{ kg}$ ,  $a = 0.1 \text{ m}$  along the pendulum.

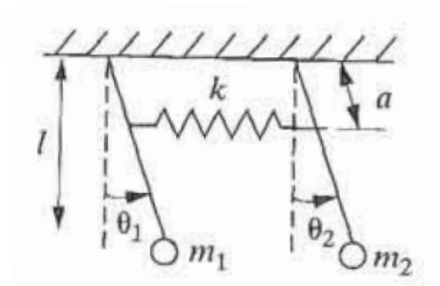


Figure 1. Two Pendulums of the same length connected by a spring used to model a machine part.

2. Find the response of the system shown in Figure 2 by modal analysis for the case where the rods have equal stiffness (i.e.,  $k_1 = k_2$ ),  $J_1 = 3J_2$ , and the initial conditions are  $x(0) = [0 \ 1]^T$  and  $\dot{x}(0) = 0$ .

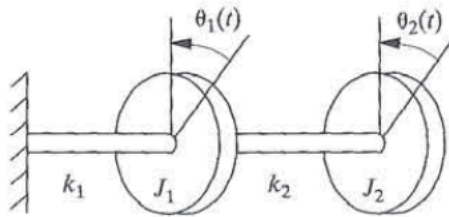


Figure 2.

3. A slightly more sophisticated model of a vehicle suspension system is given in Figure 3. Calculate the natural frequencies for  $k_1 = 103 \text{ N/m}$ ,  $k_2 = 104 \text{ N/m}$ ,  $m_2 = 50 \text{ kg}$ , and  $m_1 = 2000 \text{ kg}$ . Suppose that the tire hits a bump which corresponds to an initial condition of  $x(0) = [0 \ 0.01]^T$  and  $\dot{x}(0) = 0$ . Use modal analysis to calculate the response of the car  $x_1(t)$ . Plot the response for three cycles.

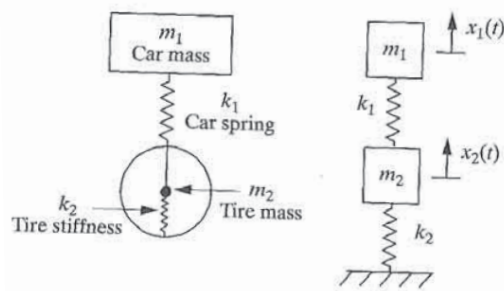


Figure 3.